Baker's Rocketship And The Mathematical Nominalist's Real Problem With Physical Magnitudes

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Abstract

Nominalists face classic Quine-Putnam indispensability worries about how to logically regiment physical magnitude statements. Measurement-theoretic uniqueness theorems offer a popular tool for answering this challenge, by showing that the application of finitely many atomic relations between non-mathematical objects can pin down unique intended values for Platonist physical magnitude functions. However, recent discussions of Baker's rocketship argument for absolutism about physical magnitudes raise a prima facie worry – that such nominalist paraphrases can only express comparativist friendly content, and therefore cannot capture the intuitive determinism of Newtonian mechanics. In this paper, I'll develop that worry and argue that even nominalists who accept Baker's premises and argument can address it by adjusting traditional nominalist paraphrases to employ absolutist physical magnitude predicates taken to be rigid designators that fix absolute scale without reference to mathematical objects.

1 Introduction

Recent work in potentialist set theory has shown that we can take a purely modal-logical perspective on pure mathematics, understanding mathematical claims as statements about logical possibility rather than as ontological claims about abstract objects [11, 13]. As Putnam notes[19], in many contexts we can seemingly equally well take either a modal or a Platonistic perspective on mathematics. Indeed, certain puzzles concerning the Burali-Forti paradox and the height of the hierarchy of sets appear to favor a modal approach to pure higher set theory[1].

This raises a natural question: can we extend this modal perspective to applied mathematics? If so, we could understand all mathematical content - both pure and applied - within a unified modal-logical framework.

However, extending the modal approach to applied mathematics requires answering classic Quinean and explanatory indispensability challenges. In particular, we must show that we can adequately logically regiment physical magnitude statements - claims about length, mass, velocity, etc. - without quantifying over mathematical objects.

In §2, I review recent work on nominalizing physical magnitude statements using measurement-theoretic uniqueness theorems and modal if-thenism [5]. These strategies appear to answer classic explanatory indispensability challenges for advocates of the modal-logicist project (though perhaps not in ways that would satisfy all forms of nominalism). In §3 I show how considerations from Baker's rocketship argument [1] (and recent discussions of absolutism vs. comparativism about physical magnitudes) threaten this nominalization project.

Roughly speaking, the problem is that Baker's arguments suggest that adequate statements of Newtonian mechanics must capture absolute scale, not merely scale-free ratios. But the measurement-theoretic paraphrases reviewed in §2 appear to capture only ratio information. This threatens their ability to logically regiment deterministic physical theories.

In §4 I argue that this worry can be defused. Even a nominalist who completely grants the success of Baker's arguments for absolutism (and the truth of the assumptions about e.g. Laplacian determinacy) can create nominalist paraphrases appropriate to their view of metaphysics by making use of the "cheap tricks" modal if-thenist paraphrase strategy from [5]. The key move is to introduce finitely many 'ugly' physical

[1]Aside from these motivations coming from the philosophy of set theory, see Bueno [6] and Berry[4, 2] for proposed answers to access worries about logical possibility which suggest that there would be epistemic benefits to showing the viability of a unified modal approach to mathematics (if the latter would let us reduce mathematical access worries to access worries about knowledge of logical possibility).

magnitude predicates and relations - such as “is one meter long” or “has mass in kg equal to the ratio between these two paths’ lengths” - and treat them as rigid designators that fix absolute scale.

Overall I aim to defend nominalists’ ability to answer classic Quinean and explanatory indispensability worries about physical magnitude statements[2]. However we will see that my proposals tend to support claims from [5] that such nominalists’ real challenges concern more metaphysically controversial reference and grounding worries rather than basic indispensability arguments. Accordingly, I don’t claim to completely vindicate the modal-logicist perspective on applied mathematics referenced above - only to clarify where the genuine obstacles lie.

Also note that my proposal about how mathematical nominalists (i.e., rejectors of mathematical objects) can address indispensability worries might not be acceptable to all mathematical nominalists. I’m specifically interested in mathematical nominalism driven by the kinds of mathematics-specific considerations referenced above (e.g. the Burali-Forti paradox in set theory and logicist intuitions that a modal-logical perspective on mathematics should be possible) and I have tailored my paraphrases to fit that project[3].[4]

[2]Further Quinean worries may remain about whether/how nominalists can logically regiment certain specific kinds of physical magnitude claims like probability statements, which seem to involve the attribution of physical magnitudes *to* objects nominalists might not be comfortable with.

[3]For example, philosophers who reject mathematical objects as part of a general physicalist, materialist, naturalist or ontologically and ideology minimizing ‘desert landscape’ project might (or might not) regard my paraphrases’ use of a primative notion of logical possibility as unacceptable. My project aims to clarify whether mathematical content can be recast in modal-logical terms, not vindicate any of these grand philosophical projects which have traditionally motivated mathematical nominalism.

[4]Note that the question of realism vs. nominalism about mathematical objects is distinct from the question of realism vs. nominalism about physical magnitudes (see Wolff [24]). One could, for instance, be a ”vigorous Platonist” about physical magnitudes—believing in mass properties or objects to which physical things are related—while remaining a nominalist about mathematical objects (numbers, sets). Such a view would eliminate quantification over numbers while retaining quantification over physical magnitude abstracta. Thus, defenders of mathematical nominalism need not defend nominalism about physical magnitudes; they need only show that attractive metaphysical positions on physical magnitudes (whether realist or nominalist) allow for logical regimentations that avoid quantifying over mathematical objects.

2 Background and Setup

Let me begin with some background on how I will understand the classic Quine-Putnam indispensability argument, and the kind of modal-if thenist paraphrase project I aim to defend.

In §2.1 I'll clarify how I'm thinking about classic explanatory indispensability arguments. I'll then motivate and introduce a key modal notion (the conditional logical possibility operator) and discuss how we can use it to flesh out a basic modal if-thenist nominalization strategy[5].

In §2.2 I'll review how appeals to measurement theoretic uniqueness theorems promise to help us implement this strategy in the case of physical magnitude statements. I'll also explain the sparse magnitudes problem referenced above and briefly outline the 'cheap tricks' proposal for solving it made in [5].

2.1 The Paraphrase Project

What does it mean to succeed in nominalistically paraphrasing our best physical theories in response to Quinean and explanatory indispensability challenges? Much could be said on this topic. But, for the purposes of this paper, I'll consider whether we can provide a nominalistic paraphrase strategy T which produces a translation $T(\phi)$ of our total best physical theory ϕ, with the following pair of good features.

1. *Truth-conditions:* $T(\varphi)$ should be true at exactly the same metaphysically possible worlds as φ, according to the Platonist. In this sense it captures all of φ's non-mathematical content.

2. *Explanatory adequacy:* $T(\varphi)$ should preserve (at least) the intuitive explanatory power of φ with respect to physical facts.

[5] cf. [11, 19]

How might one try to provide such a paraphrase? An if-thenist approach to nominalistically paraphrasing mathematical claims has enjoyed enduring popularity, going back through Hellman, Putnam, Russell and perhaps even Aristotle. Modal forms of if-thenism, as developed by Putnam and Hellman[12, 19] deploy the following basic ideas.

- First, we come up with a sentence D which completely pins down the behavior of pure and applied mathematical structures the Platonist (but not the nominalist) believes in. For example, in the case of the natural numbers some version of the second order Peano axioms might play this role.[6]
- Then we nominalistically formalize each scientific theory ϕ with apparent quantification over mathematical objects as saying something like.

  Necessarily **if** D then ϕ.[7][8]

So, basically, the modal if-thenist strategy is to have $T(\phi)$ assert that ϕ would be true if we supplemented the non-mathematical world with the mathematical (and applied mathematical) objects whose existence the Platonist assumes.

But how do we implement this strategy? A crude approach might use a counterfactual like the following.

$$T(\varphi) \ := \ D \mathbin{\Box\!\!\rightarrow} \varphi.$$

Here D is a categorical description of all the extra mathematical structure the Platonist posits (e.g. the natural numbers via second-order Peano Axioms). So the result-

[6] See [5] for a demonstration of how to replace second-order quantification with the conditional logical possibility operator.

[7] In cases where we have a categorical description of the relevant structure (i.e., any two structures satisfying the description would have to be isomorphic to each other), this gives bivalent truth conditions for all pure mathematical statements. Note that when it's necessary to use second-order quantification to pin down a categorical conception of the relevant structure, we can do this purely in the language of conditional logical possibility as discussed in [3].

[8] We may also want our nominalist paraphrase to include the claim that it is logically possible for the relevant Platonist claim to be true[11, 5]. In this case, our paraphrases will take the following form $T(\phi)$: $\Box(D \to \phi) \land \Diamond D$.

ing paraphrase of a claim like 'there are a prime number of rats' becomes something like '*if there were* objects satisfying [some categorical description of the intended structure of the natural numbers and functions from physical objects to natural numbers] then there would be a prime number of rats (i.e., there would be a 1-1 onto function pairing the rats with the numbers up to some prime).' Where suitable axioms D (pinning down the exact structure the Platonist takes their relevant universe to have at each metaphysically possible world) can be found, this strategy promises to replicate the Platonist's truth-conditions without committing us to numbers, sets, or functions as entities[9] .

However critics have objected to the above crude if-thenism's commitments regarding counterfactuals about the existence of mathematical objects (or objects satisfying axioms capturing the intended structure of mathematical objects). Can a nominalist legitimately assume that the physical/nominalistic facts they want to describe (e.g. existence and distinctness facts about) would remain fixed in the closest metaphysically possible worlds where mathematical objects exist (or infinitely many non-mathematical objects satisfy the axioms of second order Peano Arithmetic when considered under some non-mathematical relations)? If not, the above counterfactual based version of if-thenism fails.

Luckily however, such controversial assumptions about counterfactuals turn out to be unnecessary to implementing the paraphrase approach described above. We can avoid them by following [5] in using a *conditional logical possibility* operator[10]. Intuitively, $\Diamond_{R_1,\ldots,R_n}\varphi$ says: there is some way of expanding the domain and interpreting

[9]Nominalists who don't like commitment to the actuality *or the metaphysical possibility of mathematical objects* can avoid this by formulating D to assert that some other surrogate objects and relations not mentioned in the sentence to be paraphrased satisfies the axioms pinning down the intended structure the Platonist takes their mathematical objects to have, as discussed in the appendix. However trying this move when talking about counterfactuals and metaphysical possibility rather than logical possibility intensifies the worry that the closest worlds to the actual one where D is true might be ones where the physical objects we want to talk about (e.g. the rats) behave differently.

[10]See [11]'s use of the actuality operator when cashing out modal if-thenist conditionals (to be discussed in more detail below) and [14, 15] for a formally different (but I think somewhat similar in spirit) approach to solving the same problem.

additional relations such that φ holds, while holding fixed the structural facts about the relations $R_1, \ldots, R_n$. This allows us to talk about what is logically possible *given the actual structure of how certain nominalistically acceptable relations apply*.

To motivate this notion, consider a situation where there are three cats and two blankets. Could it be that each cat is sleeping on a different blanket? No, as per the pigeonhole principle. In this situation, there's some appeal to saying that it's logically impossible that each cat is sleeping on a different blanket. But, of course, it's not logically impossible simpliciter for each cat to have its own blanket. A scenario in which four cats are sleeping on four blankets is logically coherent. Rather, one might say that it is logically impossible for each cat to have its own blanket *given the structural facts about how cathood and blankethood apply*. So we seem to have a notion of logical possibility which doesn't just depend on general facts about logical combinatorics but also attempts to preserve structural features of how certain relations (e.g., cathood and blankethood) apply within a given domain.

Since this notion is distinct from plain logical possibility, we might call it conditional or structure preserving logical possibility. Using this relation lets us freeze structural facts about how non-mathematical relations apply by subscripting them[11]. And

[11] Crucially, when conditional logical possibility (CLP) claims evaluate whether ϕ is possible 'fixing structural facts', they do not merely hold fixed the truth of a set of nominalist *sentences*. Rather, they hold fixed the concrete, model-theoretic *structure* of how the relations $R_1 \ldots R_n$ actually apply in the world. In set theoretic terms, $\Diamond_{R_1 \ldots R_n} \phi$ asserts the logical possibility of ϕ in scenarios that are strictly *isomorphic to reality* regarding the extension of these specific relations, regardless of whether that physical structure can be categorically pinned down via sentences in a sparse nominalist language.

This distinction helps avoid a worry suggested by an example from Joseph Melia's [17] in 'Weaseling Away the Indispensability Argument'.

Melia shows that a nominalist theory T formulated in a language of first-order logic and mereology about a world with infinitely many mereological atoms can be complete (i.e. can imply the truth or falsehood of all statements in that language), while allowing for models that disagree about whether more complex mereological fusions —such as an infinite fusion whose complement is also infinite (e.g., the fusion of all even-numbered atoms)—exist.

Thus, a modal if-thenist cannot generally assume that it is possible to preserve the nominalistic structure of reality (or even specify all features of it relevant to the truthvalue of platonistically stateable claims like 'there is some infinite mereological fusion whose complement is also infinite') merely by requiring that all actually true sentences in an arbitrary nominalistic language remain true. As Melia's case shows, some nominalistic languages can be too expressively weak for this task.

Berry's appeal to the conditional logical possibility operator (like Hellman's [11] use of an actuality operator) avoids this threat by directly freezing the *actual model-theoretic structure* of how physical relations like parthood apply, rather than trying to achieve this effect by requiring the truth of some collection of sentences. Where Hellman puts the @ in front of some relations, Berry subscripts those relations. Thus both strategies recent philosophical work has used a conditional logical possibility operator $\Diamond_{R_1,\ldots,R_n}$ to do a few different jobs[3, 4, 2, 5] and shown that it is powerful enough to simulate second-order resources and to categorically describe standard mathematical structures like the natural numbers and various layers of an intended width hierarchy of sets. Thus, by employing it, the if–thenist can state definable supervenience conditions D that pin down the intended behavior of many pure and applied mathematical structures, while avoiding controversial assumptions about metaphysical possibility.

Accordingly, we can use this conditional logical possibility operator, we can now flesh out the basic modal if–thenist paraphrase strategy as follows.

Definable Supervenience. Suppose the Platonist vocabulary involves some mathematical relations $\vec{P}$ (e.g. 'is a set of goats') while Platonists and nominalists both agree on a finite stock of nominalistically acceptable relations $\vec{N}$ (e.g. 'is a goat', 'admires'). A *definable supervenience sentence* D is one that categorically describes how $\vec{P}$ must behave, given the facts about $\vec{N}$. Formally, D ensures that no two worlds with the same $\vec{N}$- structure could differ in their $\vec{P}$-structure. In effect, D pins down all the Platonist's extra mathematical structure in terms of finitely many nominalist relations.

Translation Schema. Once such a D is written, we translate as follows:

$$T(\varphi) \ := \ \Box_{\vec{N}}(D \to \varphi).$$

Intuitively: it is logically necessary, given the $\vec{N}$-facts, that if objects with the intended Platonist structure existed, then φ would be true. This strategy lets the nominalist

will only consider possible scenarios which are isomorphic to reality in how 'is a part of' applies, and hence (from a platonist point of view) wind up agreeing with reality on the truthvalue of 'there is some region with infinitely atoms as parts whose complement also has infinitely many atoms as part'.

I personally favor using the conditional logical possibility operator to fix structural facts about how nominalistically uncontroversial relations apply over Hellman's actuality operator approach because this approach lets us dodge controversies about quantifying into modal contexts (such as the Barcan formula) and the problem of 'metaphysically shy objects' Linnebo identifies in [13]. However, all the core problems and solutions advocated in this paper apply equally to Hellman style developments of modal if-thenism, so I won't say anything more about this preference here.

mimic Platonist reasoning without ontological commitment[12]

Arguably this modal approach aligns well with the role of modal notions in actual scientific practice: explanations of why a map cannot be three-colored, for instance, are naturally expressed modally rather than by talking about what sets or second order objects exist.

But how widely can this paraphrase strategy be applied?

2.2 Sparse Magnitudes and "Cheap Tricks" (Review)

In this section I'll briefly review some special challenges for applying the above paraphrase strategy (and similar ones) to paraphrase physical magnitude statements, and Berry's[5] "cheap tricks" argument that the nominalist's real problem with physical magnitude isn't an explanatory indispensability worry, but rather concerns accounting for facts about grounding and perhaps reference.

A classic counting argument in Putnam's [20] in effect raises a general worry about whether it is possible to nominalistically paraphrase Platonist scientific theories about physical magnitudes in any finite human learnable language. Specifically, as applied to the basic modal if-thenist paraphrase strategy above, Putnam's challenge amounts to a worry about whether any definable supervenience definition D using only finitely many nominalistic relations could precisely pin down the intended behavior of the Platonist's 'is...meters long' as needed to nominalize scientific theories like Newtonian mechanics[13][14].

[12] See appendix B for a simple example of a paraphrase using this approach.

[13] Putnam's counting argument in [20] notes that a Platonist theory of gravity can appeal to a mass-ratio relation to distinguish infinitely many physical possibilities (e.g., distinguishing a universe where the mass ratio is r from one where it is r', for any real numbers). In contrast, any nominalist language using only finitely many relations can only distinguish finitely many possibilities for a two-object universe. Thus, it appears impossible for any finite nominalistic theory (including ones of the form above) to capture the full expressive power of Platonist theories. In particular, one might worry about whether any definable supervenience sentence can be written which adequately pins down the behavior of physical magnitude functions in terms of facts about how finitely many nominalistically acceptable relations $N_1 \ldots N_m$.

[14] It's true that modal if-thenist paraphrasers don't aim or need to remove talk of mathematical objects from scientific theories in the way that straightforward Fieldian nominalizers do. They think it's enough to avoid ontological commitment by writing something like the $\Box$ [If there were objects satisfying our conception of mathematical structures (alongside all the ordinary physical objects, with all relations between these physical

Many nominalists have responded to Putnam's challenge by noting that measurement-theoretic uniqueness theorems allow us to use a small number of ratio relations to write a sentence D which uniquely pins down precise values for physical magnitude functions (up to multiplication by a constant) —at least in the case of length, if we are willing to make certain metaphysical assumptions[15]. Given suitable assumptions about

objects being kept the same) then such-and-such scientific theory would be true]

However, (as Hellman and Berry both acknowledge) model if-thenists still need to fill in the antecedent of the above conditional, articulating *what our conception of the relevant mathematical structures is*. That is, they must write down axioms which categorically pin down (and in a sense implicitly define) the intended structure of all mathematical vocabulary (relevant to scientific theories being paraphrased) given only antecedently fixed reference for logical vocabulary and finitely many non-mathematical predicates and relations.

- In the case of pure mathematical structures like the natural numbers, this is easy enough to do (assuming your logic has something like second order quantification). For you can cash out the antecedent 'there are objects behaving like the numbers' by writing down a version of the Second Order Peano Axioms (a version the first order Peano axioms which replaces the infinitely many instances of the induction schema with a single second order induction axiom). These axioms categorically describe the intended natural number structure (any two models satisfying them must be isomorphic).
- But what about specifying the intended behavior of impure mathematical structures like physical magnitude functions (e.g. a two place length in meters, or mass in grams relation)?

  This challenge is where the lineage of worries discussed in this paper (e.g., about whether any finitely learnable nominalist language could draw enough distinctions) beset the modal if thenist – just as much as more straightforward nominalist paraphrasers. The difference is just that modal if-thenists seek principles categorically pinning down the *supposed application of mathematical vocabulary* (as needed to fill in the antecedent of if-thenist conditionals), rather than trying to directly paraphrase a target physical theory.

  In connection with this project of specifying the intended behavior of physical magnitude functions, modal if thenists can worry (with Putnam) that no finite nominalist vocabulary could draw enough distinctions to pin down the intended behavior of a length in meters function. So they might naturally seek aid from measurement theoretic uniqueness results (showing that e.g. merely requiring a supposed length in meters function respect a small number of relations between physical objects and satisfy certain axioms suffices to pin down the behavior of this function up to multiplication by a constant). And (even with this aid) they will face lingering worries about sparsely instantiated magnitudes and need to express absolute physical magnitude facts (as opposed to merely ratios) in response to Baker's argument become relevant.

Note that the above challenge of categorically describing the intended behavior of mathematical structures using logical and other non-mathematical vocabulary is not a problem for everyone. A sufficiently bold Platonist could say that that we can secure reference to things like the hierarchy of sets or the mass in grams function in some way e.g. by recognition procedures or directly benefitting form reference magnetism (and thereby make claims whose truthvalue reflects whatever structure these objects happen to have) that doesn't require categorically pinning down this structure using only logical primitives. They could say that such a categorical description implicitly defining mathematical structures is not needed to make claims whose truthvalues reflect such structures, any more than a categorical description of the behavior of Abe Lincoln (in other terms) is needed for us to make biographical claims whose truthvalue depends on this behavior.

[15]Hellman tentatively discusses a few options for holding fixed non-mathematical reality for the purposes of stating modal if thenist paraphrases in *Mathematics Without Numbers*

First Hellman considers using a primitive modal operator which is said (without further explanation) to hold fixed material/non-mathematical reality. I take the appeal of avoiding introducing such a new primitive (or giving more explanation if we can) if we can to be clear.

Second Hellman considers using an actuality operator @ to hold fixed the actual world applications of finitely many relations $R_1 \ldots R_n$ when writing the antecedent of our modal if-thenist paraphrases.

space (closure under concatenation, Archimedean properties, etc.), a nominalist willing to countenance things like length relations between physical paths can show that any admissible "length function" (i.e., any one that assigns numerical lengths in a way that respects how a small number of special length ratio relations apply) must be unique up to multiplication by a positive constant.

Thus we can expand a basic definable supervenience sentence D (which describes the structure of all pure mathematical objects and a few layers of sets with ur-elements, which the Platonist believes in) by adding a further clause which exactly pins down the intended behavior of the Platonist's length in meters function. We simply add that the Platonist's intended length in meters function is the one which assigns 1 to some canonical meter path object, and assigns values in a way that respects all the length

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$$\forall x_1 ... \forall x_{k_i} [R_i^{k_i}(x_1 ... x_{k_i}) \equiv @R_i(x_1 \ldots x_{k_i})]\}$$

Using this operator lets us talk about what is possible or necessary given the fact about how finitely many relations between non-mathematical objects actually apply.

Hellman explicitly notes that for this strategy to work in facilitating nominalist paraphrases we need to be able to find finitely many nominalistically acceptable relations $R_1 \ldots R_n$ which suffice to "determine uniquely" how matheamtical vocabulary is supposed to apply (i.e., to find what I have been calling a finite 'definable supervenience' basis of relations $R_1 \ldots R_n$ for the platonist language we want to paraphrase).

But how can such a basis be found? Hellman tentatively discusses two options.

If we proceed straightforwardly and apply the actuality operator to relations between uncontroversially nominalistically acceptable objects, like 'the segment $p_1 - p_2$ is actually congruent to the segment $q_1 - q_2$ ', we face exactly the worries (e.g., about sparsely instantiated magnitudes, pinning down absolute physical magnitudes not just ratios etc) at issue in this paper.

Hellman does briefly explore an alternative approach where the mathematical nominalist accepts infinitely many such predicates in their ontology, together with "a semantic relation of 'application' or 'true-of ' between (relational) predicates and first-order objects (particles, fields, spacetime regions, whatever)."

Then rather than separately freezing the application of the two place relation 'is heavier than'

$$x \text{ is heavier than } y \leftrightarrow @x \text{ is heavier than } y$$

and each such relation, they can write a quantified statement using three place relation 'is true of' to write down a single sentence which implies the following biconditional alongside infinitely many others.

$$\text{'heavier than' is true of } \langle x, y\rangle \leftrightarrow @ \text{ 'heavier than' is true of} \langle x, y\rangle$$

I think this is all very interesting, and has some resonance with both platonic talk of forms and an instantiation relation and Eddon's 'properties of properties' strategy in [8] of taking joint carving higher order relations applying to infinitely many mass and length etc properties to be metaphysically fundamental.

However, I also take it to be clear that such a (non-mathematically) Platonistically inflected way of cutting the Gordian knot (by accepting ontological commitment to and quantification over relations) would be very controversial among nominalists. Also technical complications arise related to reconstructing or doing without the structure you lose by treating infinitely many relations like 'x is twice as heavy as y' and 'x is 2.3 times as heavy as y' as atomic when characterizing the intended behavior of a platonist mass in grams function for the purposes of if-thenist paraphrase.

ratio relations mentioned above. So we can nominalize Platonist theories involving the length in meters function straightforwardly[16].[17]

However, as Eddon [9] and others have emphasized, this strategy faces the *sparse magnitudes problem*. Length may be richly instantiated, but other magnitudes (such as mass or charge) are not e.g. they may not satisfy the Archimedean principle. In such cases the premises for the uniqueness theorem won't be satisfied. So we have no guarantee that merely including claims that the mass/charge function respects these special finitely many ratio relations and assigns value 1 to canonical unit objects in our (would be) definable supervenience description D will let us pin down the exact intended behavior of these physical magnitude functions according to the Platonist.

In [5] Berry suggests that this difficulty can be patched well enough to answer Quinean and explanatory indispensability worries by using the following two "cheap tricks."

1. *Four-place relation.* Introduce a relation $M(p_1, p_2, o_1, o_2)$ that connects two objects (with mass) and two spatial paths, holding just in case the ratio of their masses is no greater than the ratio of the lengths of the paths. This somewhat unnatural relation, together with a canonical unit-fixing clause, allows us to pin down mass values relative to length.

2. *Holism trick.* Strengthen each paraphrase with a conjunct stating that length is

[16]Specifically, we can uniquely pick out the Platonist's intended length-in-meters function (from among all other functions from objects to real numbers) by saying it assigns length 1 to some canonical path and assigns lengths in a way that respects the following nominalistic relations:

- $\leq_L$ 'path p_1 is at least as long as path p_2'
- $\oplus_L$ 'the combined lengths of path p_1 and p_2 together are equal to the length of path p_3'. (I will say a function $l(x)$ respects $\leq_L, \oplus_L$ just if for all paths a, b and c $a \leq_L b \iff l(a) \leq l(b)$ and $\oplus_L(a, b, c) \iff l(a) + l(b) = l(c)$).

[17]The element of appeal to an intended unit fixing object is somewhat controversial (it is omitted by many measurement theoretic uniqueness based nominalization), and we will see it below that some difficulties in implementation arise.

However it attractively promises to let us pin down the exact intended behavior of physical magnitude functions –and thus capture the intended truthvalue of theories like Newtonian mechanics which appear to make different predictions depending on the absolute values (not merely the ratio) of physical magnitudes.

richly instantiated[18]—something which happens to already be implied by our current best total physical theory. This avoids cases where the uniqueness theorem would otherwise fail.

So, for example, we can uniquely pick out the Platonist's intended mass function (up to a choice of unit) by having our definable supervenience sentence D assert that this function

- assigns correct value (1) to some canonical unit object like a unit kilogram (more on this below) and
- respects the following nominalistically acceptable four place relation between two objects (with mass) and two spatial paths:
  - $M(p_1, p_2, o_1, o_2)$ which holds iff the ratio of the masses of o_1 to the mass of o_2 is $\leq$ the ratio of the length of path p_1 to the length of the path p_2.

The resulting definable supervenience sentence D will uniquely pin down the intended behavior of Platonist's intended mass in grams function m (in terms of its relationship to the length in meters function l) by requiring that it satisfies the following conditions [19].

- For all objects o_1 and o_2 and paths p_1 and p_2, $\frac{m(o_1)}{m(o_2)} \leq \frac{l(p_1)}{l(p_2)}$ iff $M(p_1, p_2, o_1, o_2)$
- m assigns the value one to some canonical object we expect to have mass one gram.

[18]When stated Platonistically, this claim that length is richly instantiated quantifies over things like spatial paths and numbers, and sets/functions from spatial paths to numbers. However it turns out to be straightforward to nominalistically represent using the basic modal if thenist techniques discussed in the previous subsection.

[19]For, note that any attempt to assign the wrong mass ratio r' to a pair of objects m_1, m_2 with mass ratio r can be ruled out by considering paths p_1, p_2 whose length ratio falls between that of r and r' and noting that $\mathscr{M}$ fails the above condition for a pair of paths such that $l(p_1)/l(p_2)$ falls between r and r'. And the existence of such a pair of paths is guaranteed by the assumption that length is richly instantiated (which, indeed, implies that length ratios are dense in $\mathbb{R}$).

Thus, we can write a sentence D which correctly pins down expected Platonist behavior for length, mass and other such physical magnitude functions – at least at all possible worlds where length is richly instantiated. But (as argued in [5]) this turns out to be all we need to create a paraphrase for any theory T which (like our current best total scientific theory) implies that space is richly instantiated. So we can solve the sparse magnitudes problem.

Admittedly, these devices are quite unattractive when regarded as responses to certain grounding and reference challenges beyond Quinean/explanatory indispensability arguments. For the relations they invoke are unlikely to feature in an account of the fundamental structure of reality, and they do not yield a general paraphrase strategy for *all* imaginable scientific theories. But if we are only concerned with answering the classic Quine–Putnam indispensability challenge (to nominalize our *actual best scientific theory*, and literally state versions of our actual best scientific explanations) the cheap tricks appear to suffice. Accordingly, Berry argues, the nominalist's real difficulty with physical magnitudes doesn't concern answering indispensability arguments, but rather saying something plausible about what grounds physical magnitude facts (since inelegant notions like the four place relation above are not plausible grounding fundamentalia) and about how humans are able to formulate the full range of scientific theories we in fact can express.

3 Baker's Rocketship

Now let us turn to Baker's rocketship argument[1] and present the new problem for measurement theory using modal if thenist paraphrase strategies like the one above – which I aim to solve in this paper.

In this section, I'll briefly summarize Baker's core argument against comparativism about physical magnitudes. I'll then note how the same thought experiments can (especially for readers sympathetic to Baker's arguments for absolutism) seem to raise a worry for the approaches to nominalistically paraphrasing physical magnitude statements reviewed above.

In a nutshell, the relevant worry arises as follows.

- Baker's arguments about Laplacian determinism suggest that any adequate statement of Newtonian mechanics must be able to combine with facts intrinsic to a given time t to imply a unique outcome for a rocketship launch at that time – with this outcome depending on the absolute mass of objects, and not being determined by any mere mass ratio facts.

- It appears that nominalist paraphrases exploiting measurement theoretic uniqueness theorems which only make assertions involving physical magnitude ratio facts, like how the relations $\leq_l, \oplus_l$ and the four place relation (mass ratios to length ratios) cannot achieve this feat.

- Thus (one might fear) nominalists cannot use the measurement theoretic uniqueness theorem based 'cheap tricks' paraphrase strategy above to logically regiment Newtonian mechanics (or similar contemporary physical theories) in response to Quinean or explanatory indispensability worries.

3.1 Baker's Problem for Comparativists

Let's begin by reviewing Baker's argument for absolutism (rather than comparativism) about physical magnitudes. In [1] Baker initially explains comparativism about physical magnitudes (the target of his argument), by quoting Dasgupta's [7] as follows

> [C]omparativism is the view that the fundamental facts about mass concern how material bodies are related in mass, and all other facts about mass hold in virtue of them. (Dasgupta, forthcoming, 1)[1]
>
> [T]he comparativist thinks that the fundamental, unexplained facts about mass are facts about the mass relationships between bodies, and all other

facts about mass hold in virtue of those mass relationships. This leaves open what kinds of mass relations those fundamental facts concern: they might concern mass ratios such as an object being twice as massive as another, orderings such as an object being more massive than another, or even just linear structures such as an object lying between two others in mass. But this in-house dispute will not matter for our purposes.[7]

Exactly how to best formulate comparativism is somewhat controversial (e.g., Baker ultimately somewhat disagrees with Dasgupta's formulation)[20]. However I hope the above quotes have conveyed a decently vivid picture of the intuitions behind comparativism. And all that will matter for Baker's argument and our purposes in this paper (and something which I take all parties to agree on) is that comparativism implies the following claims:

- Any two complete descriptions of a metaphysically possible world which differ only in that all the attributed masses in one description are double (or any other positive multiple) those in the other actually refer to the same possible world.
- Therefore, any physical theory which describes any metaphysically possible world must make the same predication about scenarios which differ only in that all masses are multiplied by the same positive constant.

Baker argues that comparativism is incompatible with (one powerful but intuitive version of) the claim that Newtonian Mechanics is deterministic. Specifically, he appeals to the idea that Newtonian Mechanics intuitively makes deterministic predictions about how *different physical magnitudes trade off against one another to determine physical outcomes* in various concrete cases[21]. For example it predicts

[20] Baker proposes his own sharpened definition as follows.

> comparativism about some quantity – or family of quantities – is the view that the fundamental facts about those quantities are given by the scale-independent relations comparing different objects' values of the quantities. [1]

[21] Technically there are reasons to think Newtonian Mechanics isn't exactly deterministic. But this doesn't

- The velocity required for a rocketship to escape a planet's gravitational pull given the planet's mass.

- The acceleration of a sliding hockey puck as a function of its mass and frictional forces.

But (Baker argues) *which* outcomes are predicted by Newtonian Mechanics in such scenarios reflects the absolute values of certain physical magnitudes, not just intra-magnitude ratios. For example, there are certain pairs of mass m and velocities v, such that Newtonian mechanics predicts that:[22]

- a rocket launched from the surface of a planet with mass m at velocity v in some general situation S1 will escape

- a rocket launched at velocity v from a planet with mass 2m in a situation S2, which is exactly analogous to S1 but with the masses of all objects (the rocket, the planet etc) doubled, would not escape

This poses (Baker argues) a serious problem for the comparativist[23]–who can't allow a difference between states S1 and S2, and hence can't say that Newtonian mechanics makes unique but distinct predictions about what happens in each.

More specifically Baker invokes intuitions that Newtonian mechanics is (in certain ways[24]) deterministic, and the following 'venerable' way of cashing out determinacy.

matter to the argument. Although Baker and subsequent literature frames things in terms of capturing the intuitive determinism of Newtonian Mechanics, he could equally well made his point by talking about capturing the fact that intuitively Newtonian Mechanics requires a unique outcome for his imagined experiments with rocketships and sliding objects, and noting that comparativism has trouble doing this.

[22]Why? Newtonian mechanics says that the escape velocity for an object launched from the surface of a planet with mass m and diameter d is

$v = \sqrt{\frac{2mG}{d}}$,

where G a constant (the gravity constant). Hence it predicts if the escape velocity for the earth is v, the escape velocity for launches from a planet with the same diameter and twice the mass is $\sqrt{2}v$.

[23]My aim in this paper is to defend a view about mathematical nominalism (that nominalists can answer classic Quinean indispensability worries). So I won't take any position on absolutism vs comparativism about physical magnitudes or discuss ways of resisting Baker's arguments qua arguments for absolutism. Instead I will try to argue that *even if we entirely accept* Baker's assumption and argument for absolutism, we can still implement the cheap tricks paraphrase strategy referenced above in a way that accommodates this.

[24]Strictly speaking one might argue that Newtonian mechanics is not deterministic because it fails to make

> **Laplacean Determinism**. “A world w is [Laplacian] deterministic iff, for any time t, there is only one physically possible world whose state at t is identical to w's“[1].

Inspired by this way of thinking about determinacy, we might expect Newtonian mechanics to be deterministic in the following sense. For every momentary total state of the world S, there must be facts intuitively intrinsic to S which combine with Newtonian Mechanics to require a unique outcome for the rocket launch[25].

However (Baker argues) the comparativist cannot say this. For they only acknowledge physical magnitude ratio facts, which don't distinguish between S1 and S2. Hence they must take there to be a single momentary state S corresponding to both descriptions, and hence presumably say that two different outcomes (the rocket ship escaping or not) are equally compatible with Newtonian Mechanics plus all intrinsic facts about S. So, (Baker argues) the comparativist cannot allow Newtonian mechanics to be deterministic in the way we intuitively expect it to be.

3.2 A Challenge for Nominalist Paraphrases?

Now what about the project of nominalistically paraphrasing physical magnitude statements? I won't take a position on the truth of absolutism or whether Baker's arguments succeed in this paper.

Instead I want to point out (and then try to address) a way that accepting Baker's argument about absolutism can seem to raise a worry for approaches to nominalizing physical magnitude statements which employ measurement theoretic uniqueness theorems as discussed above[26]. Specifically, you might think Baker's arguments against

unique predictions about edge cases like Norton's dome[18] (a point mass placed at the apex of perfectly smooth and rigid dome, could remain in place or fall down in any direction). However this is no bar to Baker's argument (or the worry below). For all of these can be rephrased as appealing to the intuition that Newtonian mechanics makes unique predictions about mundane scenarios not involving idealized objects, like the rocket launch case imagined above.

[25] Strictly speaking, it's not clear that velocity facts apply to instantaneous states of the world and Baker addresses this by giving a more complicated supplementary example.

[26] cf. [22]

comparativism suggest that

- Any intuitively adequate formalization of Newtonian mechanics must (as per Laplacian determinacy intuition above) combine with statements of facts purely intrinsic to the momentary state of the world in situations S1 and S2 above to imply unique outcome to the rocketship launch in each case.

- No logical regimentation of Newtonian mechanics which only talks about intra-physical magnitude ratio relations (like the length ratio relations $\leq_L, \oplus_L$ figuring in measurement theoretic uniqueness theorems) can achieve this goal of implying distinct predictions about S1 and S2.

Thus, arguably no simple measurement theoretic uniqueness based nominalization strategy (i.e. no one that appeals only to the above mentioned ratio relations) can adequately capture the intuitive content of Newtonian Mechanics — or analogous contemporary theories relevant to Quinean-explanatory indispensability worries[27]. For no such regimentation could distinguish between total momentary states S1 and S2 (which agree on all physical magnitude ratios) – while presumably a traditional (absolutist) Platonist formulations of Newtonian mechanics could do this.

There are many possible ways of resisting the argument above. For example, nominalists attracted to comparativism could simply reject the Laplacian determinacy intuitions driving Baker's original argument, and cash out the intuitive determinacy of Newtonian mechanics in a more comparativist-friendly way [28]

[27]Technically Quinean-explanatory indispensability arguments only challenge us to logically regiment our overall best theory. And most people currently don't accept the (literal, full) truth of Newtonian mechanics as part of their best theory. So in principle nominalists could bite the bullet regarding this unwelcome conclusion while still using the style of paraphrase advocated above to answer Quinean-explanatory indispensability arguments. However, one might fear that a similar trick could be played with aspects of our actual theories which make predictions about how the state of the world will evolve forward based on absolute values of physical magnitudes rather than mere mass ratios, length ratios etc. And empirically most or all nominalists interested in straightforwardly answering Quine by providing nominalistic paraphrases would find such bullet biting about inability to capture the intuitive content of Newtonian Mechanics quite unwelcome. In the remainder of this paper, I will argue that nominalists (even those who accept Baker's arguments for absolutism) can avoid such an admission.

[28]For example one might understand the determinacy of Newtonian mechanics as only requiring there be some way of assigning lengths in SI units which respects ratio relations and makes certain formulas come out

And even nominalists who accept all of Baker's arguments for absolutism might be able to resist the worry I've raised by distinguishing two different senses in which a physical theory T might be deterministic :

- if theory T is true, then for any state S of the world at time t *the true laws of nature* only allow a unique outcome.
- *Theory T itself* combines with any complete descriptions of momentary facts about some time t to imply unique predictions about all future states.

and then saying they only accept the determinacy of Newtonian mechanics in the former sense[29]. In this way, they could resist the demand to provide a paraphrase of Newtonian mechanics with the kind of predictive power demanded by the objection above.

However, I won't try to further develop and assess such strategies in this paper.

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true (rather than attempting to fix unit objects as my proposal does) can likely still say that Newtonian mechanics has this weaker determinacy property.For the requirement to assign SI units in a way that preserves the truth of Newtonian mechanics, combined with the need to honor ratio facts, will constrain unit choices enough that all acceptable applications of SI units yield the same physical predictions. One could even motivate this weaker understanding by arguing that appeals to instantaneous velocity and mass are problematic because velocity is only defined over an interval ϵ.

Baker acknowledges this issue and writes that "this argument's soundness depends in an interesting way on a metaphysical premise: it only works if velocity is taken to be a truly instantaneous quantity, and not a property of infinitesimal temporal neighborhoods. In §4 I show that this means the comparativist should accept a theory of motion, like the popular "at-at" theory, according to which there are no truly instantaneous velocities. One can also construct more involved examples in which even an at-at theorist should agree that comparativist physics gives indeterministic predictions where absolutist physics does not. In some such cases, comparativist physics exhibits a peculiar sort of temporal action at a distance: the state of the entire past may be sufficient to determine the future, while the present state is insufficient. (In other cases, even the whole past history is insufficient.) These examples are explored in §5. They have the flavor of idealized toy models, so I hesitate to count them as evidence against the truth of comparativism. But it seems to me that determinism and temporal locality, in these cases, should not be ruled out as metaphysically impossible. Consequently, if comparativism is true, it is contingently true. "

[29] Someone who takes this perspective might say it is merely due to the Russellian 'accident' of our being unable to speak a language with infinitely many different atomic physical magnitude predicates that we can only believe a form of Newtonian Mechanics which either underdetermines the outcome of Baker's rocketship type scenarios, or only determines it when combined facts about length and mass ratios between the objects at the relevant time t and earlier times. But that accident doesn't constrain what fundamental modal facts about physical laws could be like. The actual laws of nature (if Newtonian mechanics is true) might be equally or more illuminatingly captured by a statement of Newtonian mechanics in an infinite language with uncountably many different atomic physical magnitude predicates corresponding to different physical magnitudes. Such a language could combine with instantaneous facts about the world at time t to imply a unique outcome to the Baker's rocketship case, even if nominalistic statements of physical hypotheses in human language cannot do this. On the idea that we shouldn't expect absolutist physical magnitude facts to be expressible in certain ways c.f. [16] and [23] .

Instead I will argue that even the nominalists most threatened by the above worry (ones who accept absolutism about physical magnitudes, Baker's version of the determinacy of Newtonian mechanics *and* the further demands on the predictive power of acceptable regimentations of newtonian mechanics evoked above) have a straightforward reply. Specifically, it turns out that making a few tweaks to the cheap tricks strategy above (involving rigid designators) lets us produce a nominalistic paraphrase of Newtonian mechanics which combines with facts purely intrinsic to the momentary states of the world S1/S2 (as such an absolutist would understand them) to imply a unique outcome whenever Newtonian Mechanics intuitively does.[30]

3.3 The Cheap Tricks Proposal Alone

In one sense Berry's cheap tricks proposal already attempts to avoid the worry above. For (unlike the simple modal if thenism sketched above), it *doesn't* only use physical magnitude ratio relations to characterize the intended behavior of physical magnitude functions up to multiplication by a constant. Instead it tries to capture the exact intended behavior of Platonist physical magnitude functions at each possible world w

[30]Specifically, we will produce definable supervenience sentences D with the following property (which I express using set theory though it could be expressed using the language of conditional logical possibility).

Any two models that agree with the application of nominalistic properties and relations (as absolutist understands them) between objects at time t and satisfy our definable supervenience description D will agree on the physical magnitudes in meters, kg etc. assigned to all objects at time t, and hence agree on the truthvalue of Platonist applied math conditionals like 'if Newtonian mechanics is true then the rocketship will fall back towards the planet'.

Plugging the resulting paraphrases into the modal if-thenist paraphrase strategy above will let us vindicate the claim that 'Newtoninan Mechanics combines with facts intrinsic to time t to determine a unique outcome' in two senses.

For each possible world and situation S1/S2 of the kind imagined above, some modal conditional of the following form holds: It's logically necessary given the facts about how nominalistically acceptable relations apply at time t (something expressible using the basic conditional logical possibility operator with a few tricks) that if there are objects satisfying Platonist axioms and Newtonian mechanics and the characterizations of physical magnitude functions at t then the rocketship will never fall back within n radius of the planet'.

Each claim which a Platonist absolutist would regard as an example description of facts intrinsic to the state of the world at time S (e.g. 'the planet has such and such mass in kg and length in meters and the rocketship has mass in kg') which combines with the Newtonian mechanics to imply a precise prediction about whether the rocket escapes will be paraphrased by a nominalistic statement which the absolutist would regard as is equally intuitively intrinsic to facts at time t and equally combines with the corresponding nominalization of Newtonian mechanics to imply a unique predicted outcome.

(where length is richly instantiated) by requiring that any such functions assign specific values to 'canonical unit-fixing objects'. For example, its proposed definable supervenience sentence D attempts to characterize the intended behavior of the length in meters function (within the larger structure of objects Platonists believe in), by requiring that this function (i) respects the length ratio relations $\leq_L$ and $\oplus_L$ and (ii) assigns value 1 to the/all canonical meter-length object(s). It thereby attempts to capture exactly the possible world truth conditions (and hence the physical predictive consequences) of any Platonist logical regimentation of Newtonian Mechanics you feed into it.

However, problems arise when we think more carefully about what relevant 'canonical unit fixing objects' could be. One problem is this. The Laplacean determinacy challenge above §3.2 requires that our paraphrase of Newtonian mechanics imply precise outcomes to the rocketship launches in S1 an S2, given only facts about *properties and relations holding between objects at that time (whose application is intuitively intrinsic to that time).* However, the canonical unit fixing objects referenced above might not exist at the instant in question. So the basic cheap tricks proposal might yield paraphrases that only succeed in pinning down unique mass in kg values for the rocketship and planet – and thereby predicting a unique outcome of launch – given facts which are extrinsic to the state of the world at the time of launch (namely facts relating the planet and rocketship to the canonical unit fixing object). Worries also arise about whether any choice of canonical unit fixing objects will give our paraphrases appropriate modal behavior[31]. To see what I mean concretely, consider the following options for what 'canonical unit fixing objects' could be:

Historical artifacts: If we fix the meter by reference to historical artifacts like the meter stick in Paris, then our paraphrase of Newtonian mechanics will only make de-

[31] Recall that our victory conditions for successful Quinean/explanatory paraphrase required producing something Platonists must regard as having (not just the same actual world predictive implications) but the same possible worlds extension as Platonist formulations of our total best scientific theory. And note that our cheap tricks strategy above attempts to create a definable supervenience sentence D which the Platonist would regard as correctly and categorically describing the intended behavior of all extra vocabulary at all metaphysically possible worlds where length is richly instantiated.

terminate predictions about Baker's rocketship scenarios S2 and S1 when facts about relations between objects at time t are combined with extrinsic facts about relations to the canonical meter stick (which might not exist at the time of the launch being considered). Moreover, such an approach would implausibly make the length of the Paris meter stick metaphysically necessary, and make our paraphrases of scientific sentences trivial or meaningless in possible worlds where no such stick exists.

Physically Well Behaved Objects and Processes: Modern definitions of SI units may lessen the modal worry above by defining units through appeal to enduring physically necessary regularities—for instance, defining the second via the radiation frequency of cesium-133 atoms in a specific transition, then defining the meter via the distance light travels in a specified fraction of a second. This may help with the modal worries referenced above. For perhaps it is less implausible that these unit fixing objects have relevant physical magnitude properties metaphysically (or at least physically[32]) necessarily. The resultant nominalist paraphrases might even be attractive to comparativists about physical magnitudes[33]. However, this approach still fails to satisfy the Laplacian determinacy demand above. For to fix scale this way, our definable supervenience description D must reference events and processes (cesium atom transition, light travel) that are not part of the momentary state of the world in situations S1 and S2 at the exact time of launch. Thus it may only imply unique predictions about outcomes when combined with facts about relations extrinsic to the time under consideration.

So I think it's prima facie unclear whether Berry's [5] breezy advice to use 'canonical unit objects' to fix the scale of physical magnitude functions at each possible world

[32] If we knew this, for some target physical theories might be able to try a version of the second 'cheap trick', arguing that we only need to get physical magnitude facts right at possible worlds which aren't guaranteed to make our paraphrase of our best physical theory false for other reasons.

[33] Such comparativists might say that, given the meaninglessness of cross possible world mass/length etc comparisons, all that can be meant by apparently absolutists physical theories which attempt to say that mass and charge trade off against each other in a certain way when determining the outcome of physical events is that, as a matter of physical law, physical magnitudes trade off against each other in a way that reflects the relations they bear to canonical fixing object objects/events types (which must always have a fixed value of the constant they define) like the cesium atoms and light rays mentioned when specifying SI units.

w (and thus capture correct truth possible worlds truth conditions) can be implemented. And there special concerns about whether this can be done in a way that's acceptable to absolutists (who, in a sense, take there to be more metaphysically possible worlds), and satisfies the strong Laplacian determinacy demand above.

4 Addressing Baker's Rocketship Inspired Worries for Nominalist Paraphrases

4.1 Core Proposal

Happily however, we can solve both problems by making a few small modifications to the basic cheap tricks proposal above, involving rigid designators – or so I shall argue. In particular, I will show how we can write a definable supervenience description for physical magnitudes which absolutists could accept as

- pinning down correct scale for physical magnitude functions at all metaphysically possible worlds *w* where length is richly instantiated
- doing this, for each time *t*, in a way that is sufficiently intrinsic to the state of the world at time *t* to satisfy the Laplacian determinacy intuitions above

The key idea will be that, rather than plugging descriptions of canonical unit fixing objects directly into our definitions, we can regard human practices involving such objects as fixing the meaning of a small number of *atomic*[34] *physical magnitude predicates and relations* — like 'is one meter long' and relations like 'has mass in kg equal to the ratio between these paths' lengths'— which we take to be rigid designators, and apply widely enough to pin down intended scale for all physical magnitude functions

[34]That is, these predicates are regarded as conceptually primitive for the purposes of logical formalization capturing the logical structure of our thought and talk. Just as with the 'ugly' four place relation referenced in the original cheap tricks proposal, we need not regard such notions as metaphysically fundamental (c.f. Sider on different kinds of analysis in [21]).

at a given time t by reference only to objects existing at t. This allows us to fix scale correctly (from an absolutist point of view) at all metaphysically possible worlds[35]. And for each time t, this will allow us to pin down the behavior all physical magnitude functions at that time using only facts about the instantaneous state at time t, without requiring that any particular kind of object we regard as canonical and meaning fixing (e.g. a cesium atom or the canonical meter stick) exists at time t.

Specifically, I claim we can fix an intended scale for physical all magnitude functions by appealing to *finitely many nominalistically acceptable properties and relations which are taken to be rigid designators*, as follows.

- To pin down a unique intended length in meters function, introduce an atomic predicate 'is one meter long', whose meaning is given by connection to the results of counterfactual measurement procedures (if performed in the actual world) rather than reference to a fixed object like a meter stick.
  - Take this term (and the atomic three-place relation term proposed below) to be rigid designators – expressing a certain absolute length property which is had by canonical unit fixing objects (e.g. paths traveled by light, the meter stick in Paris) in the actual world.
  - Have your supervenience description D fix units for length, by requiring that an intended length-in-meters function must assign all spatial paths which have the 'one meter long' property the value 1.
  - That is, pick out the intended length-in-meters function l, by requiring that for every spatial path p, $l(p) = 1$ iff p has this atomic property (i.e., p is 1 meter long), as well as that l respects the relations $\leq_L, \oplus_L$ (as discussed in the sections above).

[35]Or at least it lets us thus fix scale at all possible worlds relevant to applying the basic cheap tricks proposal. The strategy I propose may not let us pin down a unique intended behavior for the length in meters function in possible worlds where length is (contrary to our best total theory of the world and assumptions about space involved in Newtonian mechanics substantivally construed) very sparsely instantiated. But (as argued in the discussion of the second cheap trick in [5]), this is not a problem because our modal if thenist paraphrases will already be false at such worlds for other reasons .

- To pin down a unique intended physical magnitude function with units (e.g., mass in kg), for each of the finitely many other physical magnitudes that might occur in your theory, introduce a corresponding atomic three-place relation like 'x is as massive in kg as the ratio between paths y and z', which serves as a bridge between different physical magnitudes.
  - The reference of these atomic relations can be communicated by appeal to more convoluted counterfactual measurement procedures, but again we should take them to rigidly designate.
  - Have your supervenience description D fix units by requiring that the intended mass in kg/brightness in lumens/etc. function assign values in a way that respects this ratio relation (when considered together with an intended length in meters function whose behavior we have uniquely pinned down as above).
  - That is, pick out the intended mass in kg function m by noting that it assigns masses in such a way that for all objects x and spatial paths y and z, $m(x) = l(y)/l(z)$ iff this atomic relation applies. $\forall x \forall y \forall z[m(x) = l(y)/l(z) \leftrightarrow$ 'the mass of x compares to 1 kg as the length of path y to that of path z]'

Fixing a unique intended extension for each physical magnitude function in this way lets us address both of the desiderata above.

First (with regard to general Quinean explanatory challenge answering demands) it lets us write a definable supervenience description D which an absolutist about physical magnitudes would regard as uniquely determining intended behavior for all physical magnitudes functions (at all possible worlds where length is richly instantiated and some physical path has length 1 meter[36]). Thus plugging this description D into the modal if-thenist paraphrase strategy above lets us write down a paraphrase of Newto-

[36]Note that if these requirements are satisfied then, for every value v some physical magnitude function with units could take on, there will be some pairs of spatial paths within the path of length one meter, whose ratio gets arbitrarily close to v.

nian mechanics which gets the possible worlds truth conditions right, from a Platonist point of view (as desired).

Second, formulating the definable supervenience description D for our modal paraphrases in this way produces a version of Newtonian mechanics which satisfies the Laplacian determinism intuition evoked above. For it produces a definable supervenience description which pins down all physical magnitudes at time t, using only nominalistic relations between objects (including spatial paths) existing at time t between objects existing at time t. Thus the paraphrase get by plugging newtonian mechanics into our modal if-thenist structure $\Box_{N_1..N_m}(D \to \phi)$ is able to imply a unique outcome for the experiment which Baker imagines when given only facts intrinsic to the time of the experiments imagine in S1 and S2.

To see why, note that e.g. any two set models which agree on their interpretation of what nominalistically acceptable objects exist at time t and how relevant nominalistically acceptable relations apply and satisfy the definable supervenience description D proposed above, must agree on the mass in kg and length in meters of all objects existing at the time of rocket launch in situations S1 and S2, and hence require the same outcome for the rocket launch.[37].

5 Conclusion

In this paper I have defended the following perspective on how classic Quinean/explanatory indispensability worries apply to philosophers who hope to extend modal nominalist perspectives on pure mathematics (e.g. logical possibility based potentialism about higher set theory) to applied mathematics. Such nominalists can plausibly answer classic Quinean and explanatory indispensability worries about physical magnitude statements via a version of the 'cheap tricks' strategy proposed in [5] – if they are willing to employ certain kinds of finite but ugly (not plausibly metaphysically fundamental)

[37] See appendix C for how this proposal can handle some further issues about physical constants

vocabulary in their paraphrases.

Specifically, I've reviewed how Baker's rocketship argument for absolutism about physical magnitudes (as necessary to capture certain Laplacian indeterminacy intuitions about Newtonian mechanics) raises a worry that existing modal if thenist nominalization strategies like the cheap tricks approach cannot logically regiment Newtonian Mechanics in a way that captures the absolutist content Baker takes it to have. I've then proposed a wayu that would-be mathematical nominalists sympathetic to Baker can address this worry. By slightly modifying the original cheap tricks approach to use some additional ugly relations taken to be rigid designators, we can create absolutism-friendly paraphrases of Newtonian Mechanics which fully honor relevant Laplacian determinacy intuitions (by combining with facts *purely intrinsic* to a time t to imply unique outcomes for experiments at time t).

Thus I think we can indeed formulate modal if-thenist paraphrases which address Quine-Putnam indispensability challenges concerning physical magnitudes [38] (for purposes like vindicating the possibility of taking a modal-logical perspective on all mathematics). However (as noted above) worries about whether nominalists can adequately capture reference and grounding facts about physical magnitude statements may remain[39].

[38] I haven't said anything about the very important and realistically central case of probability statements, and how to paraphrase physical magnitude claims that apply to events.

[39] First, there's a reference worry. The cheap tricks proposed above can be used to paraphrase total physical theories (as needed to answer the Quinean challenge), but not more limited claims that, for example, do not include commitment to length being richly instantiated.

Second, there's a grounding and intrinsicality worry. The paraphrases I've mentioned above are crafted to get truth conditions right. But they do so by using conceptual machinery like the four-place relation in §4, which is both inelegant (and therefore implausible as metaphysically fundamental ideology) and prone to introduce intuitively irrelevant extrinsic content into paraphrases. So even if my proposal succeeds in solving the Quinean and explanatory challenges at issue in this paper, arguably further work remains to be done regarding the following projects:

- Field's ambition of stating physical laws *purely intrinsically*, rather than by talking about actual physical objects' (actual or modal hypothetical) relationship to things like objects playing the role of mathematical structures like the natural numbers[10].
- Sider's ambition of stating facts about fundamentalia (adequate to ground the truth of all other claims we believe) using *only maximally joint carving* vocabulary[21].

A Set Theoretic Mimicry

Although the conditional logical possibility operator is proposed as a conceptual and metaphysical primitive, we can use the familiar formal background of set theory to *mimic* intended truth conditions for statements in a language containing the logical possibility operator $\Diamond$ alongside usual first-order logical vocabulary (where distinct relation symbols R_1 and R_2 always express distinct relations) as follows.

> A formula ψ is true relative to a model $\mathscr{M}$ ($\mathscr{M} \models \psi$) and an assignment ρ which takes the free variables in ψ to elements in the domain of $\mathscr{M}$[40] just if:
>
> - $\psi = R^k_n(x_1 \ldots x_k)$ and $\mathscr{M} \models R^k_n(\rho(x_1), \ldots, \rho(x_k))$.
> - $\psi = x = y$ and $\rho(x) = \rho(y)$.
> - $\psi = \neg\phi$ and ϕ is not true relative to $\mathscr{M}, \rho$.
> - $\psi = \phi \wedge \psi$ and both ϕ and ψ are true relative to $\mathscr{M}, \rho$.
> - $\psi = \phi \vee \psi$ and either ϕ or ψ are true relative to $\mathscr{M}, \rho$.
> - $\psi = \exists x \phi(x)$ and there is an assignment ρ' which extends ρ by assigning a value to an additional variable v not in ϕ and $\phi[x/v]$ is true relative to $\mathscr{M}, \rho'$[41].
> - $\psi = \Diamond_{R_1 \ldots R_n}\phi$ and there is another model $\mathscr{M}'$ which assigns the same tuples to the extensions of $R_1 \ldots R_n$ as $\mathscr{M}$ and $\mathscr{M}' \models \phi$.[42]

Note that this means that $\perp$ is not true relative to any model $\mathscr{M}$ and assignment ρ.

If we ignore the possibility of sentences which demand something coherent but fail to have set models because their truth would require the existence of too many objects, we could then characterize logical possibility as follows:

[40] Specifically: a partial function ρ from the collection of variables in the language of logical possibility to objects in $\mathscr{M}$, such that the domain of ρ is finite and includes (at least) all free variables in ψ

[41] As usual (?) $\phi[x/v]$ substitutes v for x everywhere where x occurs free in ϕ

[42] As usual, I am taking $\Box$ to abbreviate $\neg\Diamond\neg$

Set Theoretic Approximation: A sentence in the language of logical possibility is true (on some interpretation of the quantifier and atomic relation symbols of the language of logical possibility) iff it is true relative to a set theoretic model whose domain and extensions for atomic relations captures what objects there are and how these atomic relations actually apply (according to this interpretation) and the empty assignment function ρ.

B Example of Basic If-Thenist Paraphrase Strategy With Conditional Logical Possibility

Example: Goats. Consider the sentence:

GOATS: "There are some goats who admire only each other."

The Platonist might regiment this as: there exists a non-empty set S of goats such that every member of S admires only other members of S. To paraphrase this, we write a definable supervenience sentence D that uniquely fixes the intended behavior of the **sets of goats** (extensionality, closure under subsets, etc.) given the 'goat' and 'admires' relations. Then our translation is[43]:

$$T(\text{GOATS}) := \Box_{\mathsf{goat},\mathsf{admires}}(D \rightarrow \exists x[\, x \neq \emptyset \wedge \forall y \in x(\mathsf{goat}(y) \wedge \forall z(\mathsf{admires}(y,z) \rightarrow z \in x))]).$$

From the Platonist's perspective, this description D (of the supposed structure of

[43] Some nominalists might worry about the above translations' use of mathematical vocabulary like 'set' and 'element' inside the $\Diamond/\Box$ of logical possibility/necessity. For, as stated, the above paraphrases make claims about how it would be logically (not to say metaphysically!) possible for there to objects like sets with ur-elements. Nominalists who think 'set' is a meaningful predicate which just happens to have a necessarily empty extension, this is fine. However, nominalists who aren't fine with this should note that we can entirely banish terms like 'set' and element from the above paraphrases, using any other first-order predicates and relations that don't occur in $\vec{N}$ instead. For example, we could uniformly replace 'set' and 'element' in the translation above with,'angel' and '...is transubstantiated into...' in our $T(\phi)$. This strategy is reminiscent of Putnam's strategy for stating potentialist set theory in [19].

the sets of goats) holds in all metaphysically possible worlds – so T has exactly the same truth conditions as $T(GOATS)$. It's true that $T(GOATS)$ picks out the relevant set of possible worlds differently from GOATS, insofar as it employs conditional logical possibility operator (talking about how the structure of the goats under admiration allows for the logical possibility of supplementation with objects satisfying certain set theoretic principles) rather than directly asserting the existence of suitable set. But this modal aspect arguably captures the intuitive meaning and the explanatory force of the natural language version of GOATS at least as well.

C Physical Constants

Technically, there is one further issue we need to address in order to logically regiment theories like Newtonian mechanics in the way I have proposed, namely how to logically regiment talk of physical constants like the gravity constant g occurring in ordinary statements of Newtonian mechanics.

This is a prima facie problem for Platonist and nominalist logical regimentations alike. How can we reconcile demands for finite statability in a human learnable language with the fact that such physical constants which (so to speak) should have infinitely many significant figures? Given any ordinary integer name like 42 in a Platonist physical theory, it is straightforward to create a corresponding definite description for use in our nominalist paraphrase. But how are we to handle talk of physical constants like the gravity constant g which have infinitely many digits, some of them unknown?

The Platonist who accepts Newtonian Mechanics can handle this issue by regarding physical constants like g as names that rigidly designate a certain real number which can only be learned by successive experiments. In this way they can write a finite statement of Newtonian Mechanics which, when combined with a complete description of the world at time t implies unique precise world state for every time after t.

But can the nominalist to come up with a similar finite, human-stateable logical regimentations of theories (like Newtonian mechanics) which seem to be stated using real number physical constants? The nominalist faces a prima faice problem. For they cannot mimic the Platonist's move above. Because they don't think there are numbers, they can't regard any name in our language as rigidly designating a number.

However the nominalist can solve this problem by repeating the trick from the previous section – i.e. by appealing to ugly (but widely instantiated) relations which we take to be rigid designators. Specifically they can replace the above Platonist's appeal to g rigidly designating a certain real number with an analogous appeal to a relations between spatial paths whose meaning we fix by associating it with the same measurement practices the Platonist takes to give meaning to g, treating (in effect) 'path ... is [actual world gravity constant] times as long as path ...' as an atomic relation which rigidly designates. Our definable supervenience description D (which is supposed to exactly describe the extra mathematical structure the Platonist believes exists in every relevant possible world) can then pick out the specific real number *r* the Platonist takes 'g' to name by saying that, e.g., for any pair of paths a and b which stand in the above atomic relation, *r* is the ratio between the number assigned to a and that assigned to b by an intended length in meters function. Note that in worlds where length is (at all times) richly instantiated, there will always be some pairs of spatial paths a and b which stand in this relation.

Overall, the puzzle of how logically regiment theories involving physical constants is not unique to nominalism - Platonists face the identical challenge. Both must explain how finite vocabulary can capture constants with infinitely many significant figures. Above I've suggested that the obvious Platonist solution - treating 'g' as a name whose reference is fixed by actual-world measurement practices and which rigidly designates that number across worlds - translates straightforwardly into nominalist terms: we just treat 'path a is g-times-as-long as path b' as a *relation* whose meaning is fixed by the same practices and which also rigidly designates across worlds. No new problems arise; the nominalist simply parallels the Platonist's already-necessary move

Thus it appears that a nominalist wiling to use 'ugly' relations which are implausible candidates for fundamentalia can paraphrase Newtonian mechanics in absolutist spirit which directly addresses the Baker's rocketship worries referenced above.

References

[1] David John Baker. Some Consequences of Physics for the Comparative Metaphysics of Quantity. In Karen Bennett and Dean W. Zimmerman, editors, *Oxford Studies in Metaphysics Volume 12*, pages 75–112. Oxford University Press, 2020.

[2] Sharon Berry. Mathematical Access Worries and Accounting for Knowledge of Logical Coherence. *The Journal of Philosophy*. forthcoming.

[3] Sharon Berry. Modal Structuralism Simplified. *Canadian Journal of Philosophy*, 48(2):200–222, 2018.

[4] Sharon Berry. (Probably) Not companions in guilt. *Philosophical Studies*, 175(9):2285–2308, 2018.

[5] Sharon Berry. *A Logical Foundation for Potentialist Set Theory*. Cambridge University Press, Cambridge, 2022.

[6] Otávio Bueno. The Epistemology of Modality and the Epistemology of Mathematics. In Bob Fischer and Felipe Leon, editors, *Modal Epistemology After Rationalism*, pages 67–83. Springer International Publishing, Cham, 2017.

[7] Shamik Dasgupta. Absolutism Vs Comparativism About Quantity. *Oxford Studies in Metaphysics*, 8:105–150, 2013.

[8] M. Eddon. Fundamental Properties of Fundamental Properties. In Karen Bennett and Dean W. Zimmerman, editors, *Oxford Studies in Metaphysics, Volume 8*, page 0. Oxford University Press, December 2013.

[9] Maya Eddon. Quantitative Properties. *Philosophy Compass*, 8(7):633–645, 2013.

[10] Hartry H. Field. *Science without numbers: a defense of nominalism.* Oxford University Press, Oxford, United Kingdom, second edition edition, 2016.

[11] Geoffrey Hellman. *Mathematics without numbers: towards a modal-structural interpretation*. Clarendon Press, Oxford New York, 1994.

[12] Geoffrey Hellman. Structuralism Without Structures. *Philosophia Mathematica*, 4(2):100–123, 1996.

[13] Øystein Linnebo. The Potential Hierarchy of Sets. *Review of Symbolic Logic*, 6(2):205–228, 2013.

[14] Øystein Linnebo. Putnam on mathematics as modal logic. In Geoffrey Hellman and Roy T. Cook, editors, *Putnam on Mathematics and Logic*. Springer Verlag, Berlin, 2018. doi: 10.1007/978-3-319-96274-0_14.

[15] Øystein Linnebo. *Thin Objects: An Abstractionist Account*. Oxford University Press, Oxford, 2018. doi:/10.1093/oso/9780199641314.001.0001.

[16] Niels C. M. Martens. The (un)detectability of absolute Newtonian masses. *Synthese*, 198(3), 2021.

[17] Joseph Melia. Weaseling Away the Indispensability Argument. *Mind*, 109(435):455–479, 2000.

[18] John D. Norton. Causation as Folk Science. *Philosopher's Imprint*, 3(4), November 2003.

[19] Hilary Putnam. Mathematics Without Foundations. *Journal of Philosophy*, 64(1):5–22, 1967.

[20] Hilary Putnam. *Philosophy of Logic*. London: Allen & Unwin, 1971.

[21] Theodore Sider. *Writing the Book of the World*. Oxford University Press, 2011.

[22] Theodore Sider. *The Tools of Metaphysics and the Metaphysics of Science*. Oxford University Press, 2020. Google-Books-ID: Gd7QDwAAQBAJ.

[23] Julien Nicolas Tricard. Playing Ozma Games with Kibble Balances: A New Defence of Absolutism About Mass. *British Journal for the Philosophy of Science*.

[24] J. E. Wolff. *The Metaphysics of Quantities*. Oxford University Press, Oxford, 2020.