Which Potentialist Set Theory?

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Introduction

Potentialist set theory promises to help us solve puzzles about the intended height of the hierarchy of sets.

However, philosophers have developed two different schools of height potentialism

- minimalist (Putnam, Hellman, Berry)
- dependence (Parsons, Linnebo, Studd)

Roughly speaking

- The **minimalist** approach interprets set theory as talking about how there could be objects (of any kind) satisfying certain set theoretic axioms.
- The **dependence** approach interprets set theory as talking about what *sets* there (in some sense) could be.

Research on these two ways of developing potentailist set theory has gone along curiously in parallel, with little discussion of reasons for favoring one appraoch over the other.

In this talk, I will try to

- clarify (and defend) the arbitrariness worries for traditional set theory that motivate potentialism
- develop 3.5 aguments for favoring minimalist potentialism, surrounding the following point.
    - Dependence potentialists appear to accept something very like minimalist potentialism about the ordinals
        - e.g. using it to explain their key modal notion of interpretational possibility
    - So if you already accept minimalist potentialism about the ordinals, why not treat set theory the same way?

Admittedly, given the broadly neo-Carnapian sympathies which many potentialists share, debates about which formal explication of set theory is 'right' might seem odd but

- I don't mean to assume that there's a unique best explication for all purposes.
- I'll argue that minimalist explications are best given certain philosophical aims and background assumptions
  - If dependence potentialists reply that they have different goals in mind, bringing this fact out will be valuable.

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Why (Height) Potentialism?

Let me start by clarify the puzzle for traditional ('actualist') set theory which I take to motivate adopting some form of height potentialism.

The Iterative Hierarchy Conception of Sets

The iterative hierarchy conception of sets avoids Russell's paradox for naive set theory by saying

- All sets exist within a hierarchy of different layers (that satisfy the well-ordering axioms).
- There's the empty set at the bottom.
- And each layer of sets contains sets corresponding to 'all possible ways of choosing' from sets generated below that layer

How Tall is the Heirarchy

So the iterative hierarchy conception gives a seemingly precise and coherent characterization of the intended width of the hierarchy of sets

- But what about its height? How many layers of sets are there?
- Thinking about this question generates a puzzle as follows...

All the way up, darn it!

Naively, it is tempting to say the following

> ***Naive Height Principle:*** *If some objects are well-ordered by some relation $<_R$, there is an initial segment of the hierarchy of sets whose structure mirrors that of these objects under relation $<_R$*
>
> - *(in the sense that the objects related by $<_R$ could be 1-1 onto order-preservingly paired with the layers in this initial segment).*

Burali Forti Paradox

But this conception cannot be correct.

- For consider the way objects are well ordered by the relation $x <_R y$ iff
  - x and y are both layers in the hierarchy of sets and x is below y
  - or x is a layer in the hierarchy of sets and y is the Eiffel tower

The Arbitrariness Worry for Actualists

So, although we seem to have a precise and coherent conception of the intended *width* of the hierarchy of sets,

- we don't seem to have a precise and coherent conception of the intended height for the hierarchy
- and it seems arbitrary to say that the hierarchy of sets just happens to stop somewhere (not determined by anything in human practice or conception)

Clarifying the Arbitrariness Worry I

- Note: the worry here isn't simply that it might be impossible to define the intended height of the hierarchy of sets in other terms.
  - After all, every theory must have some some conceptual primitives.
  - If someone claimed to have a primitive grasp of 'absolute infinity' (the intended height of the hierarchy of sets) they would avoid this problem. (But I've never met anyone who does)
- Rather it's that, once we reject the naive notion of the height of the sets, there is no obvious fall back which even pretends to pick out a unique intended height.

Clarifying the Arbitrariness Worry I

So actualist seems committed to positing an extra brute (reference magnetic) joint in nature, a fact about where the hierarchy of sets stops that is not

- motivated by anything in our conception of sets.
- independently attractive/intuitive as a natural kind
  - Claiming a primitive grasp of absolute infinity (the intended height of the hierarchy of sets) analogous to the grasp of 'all possible ways of choosing' backing intended second-order quantification would block this worry
    - but, to my knowledge, no one does.

Clarifying the Arbitrariness Worry II

Commitment to an arbitrary stopping point' worry: Why does the hierarchy of sets stop at the particular point it does[1]?

- Note: this challenge can't be answered merely by citing Russell's paradox etc. arguments against a set of all sets.
- compare the two readings of: Why am I not taller than I am?
  - Why don't I have the property of being taller than myself?
    - It's a matter of logic that no one is taller than themselves.
  - Why am I not taller in the sense of being, e.g., 5"5 rather than 5"3? Is this due to genes, early childhood nutrition, some combination etc?
    - **This** is the analog of the arbitrary height question for the actualist.

[1] rather than going through another limit ordinal

Avoiding Arbitrariness With Potentialism

Potentialist explications of set theory responds to this problem by

- rejecting the idea of a unique intended point at which the hierarchy of sets stops (a single intended structure of the sets up to isomorphism)
- reinterpreting ordinary set theoretic statements, to replace
  - apparent quantification over a single intended hierarchy of sets
  - with claims about how it would be (in some sense) possible for intended-width initial segments of the hierarchy of sets (or objects with the structure thereof) to be extended.

However, the above basic idea has been fleshed out in different ways.
Following Barton I will call these **minimalist** and **dependence theoretic** potentialisms.

Now let's go into a bit more detail about each kind of potentailism

- starting with minimalist potentialism (they style I'll be advocating in this talk)....

Minimalist Potentialism

In [11] Putnam suggests a modal perspective where set theory considers what ‘models’ of set theory are, in some sense, *possible* and how such models can be extended. He

- considers ‘standard’ models of set theory built of concrete objects, e.g., pencil dots that are related by physical arrows.
- suggests that we can understand set-theoretic statements as claims about what such concrete models are possible, and how they can be expanded.

Minimalist Potentialism

Example: We can paraphrase a set theoretic statement of the form '$(\forall x)(\exists y)(\forall z)\phi(x, y, z)$' where ϕ is quantifier free, as saying:

> *if G is a concrete model, and p is a point within G, then it is possible that there is a model G' which extends G, and a point y within G' such that necessarily, for any concrete model G'' which extends G' and contains a point z, such that $\phi(x, y, z)$ holds within the concrete model G''.*

In influential early work, Hellman sharpens and develops this proposal by understanding

- the relevant notion of possibility $\Diamond$ as logical possibility
  - approximately interdefinable with entailment
  - something we have independent reason to take as primitive rather than cashing out using set theory (c.f. Field and others on this point [6, 7, 4, 5])
- 'standard models' as models which (basically) satisfy ZFC_2.
  - But Hellman later explores other options, and the version of minimalist potentialism I favor [2, 3] replaces:
    - second order quantifiers with appeal to a notion of of structure-preserving 'conditional logical possibility' $\Diamond_{...}$ (see next slide)
    - ZFC_2 with axioms IHS that merely express our iterative hierarchy conception (no claims about height)

Conditional Logical Possibility, Very Briefly

If a physical map is not three-colorable we might say:

$\neg\Diamond_{adjacent,country}$ [Every country is either yellow, green or blue and no two adjacent countries are the same color]

'It's logically impossible, given the (structural) facts about how 'is adjacent to' and 'is a country' apply on the map above, that every country is either yellow, green or blue and no two adjacent countries are the same color.'

Notably, all versions of minimalist potentialism eliminate talk of sets and elementhood, replacing it with e.g.,

- second-order quantification 'It's logically necessary that $(\forall X, f$ if $ZFC_2[set/X, \in /f]$ then...)'
- non-mathematical one and two place relations 'It's logically necessary that if the ink dots and arrows satisfy ZFC_2 then..'

So Hellman's minimalist paraphrase of "$(\forall x)(\exists y)(x \in y)$" looks like

$$\Box(\forall V_1)(\forall x)[x \in V_1 \rightarrow \Diamond(\exists V_2)(\exists y)(y \in V_2 \wedge V_2 \geq V_1, \wedge x \in y)]$$

(where quantification over all V_i is shorthand for quantification over all second-order objects X, f satisfying some axioms like ZFC_2)

Dependence theoretic potentialism

In contrast, dependence theoretic potentialists

- acknowledge the existence of special objects called 'sets'
- but interpret set theory potentialistically, as talking about what sets **could be formed** (where the fact that these are sets plays an essential role not captured in mere axioms)
- In what sense of 'could be'?
  - the idea that there could (in some sense) be more sets than there actually are can initially seem mysterious.

The dependence potentialists I will focus on (like Linnebo[9, 8, 10] and Studd[12])

- accept that all pure sets exist *metaphysically* necessarily.
- cash out potentialist set theory by appeal to a notion of **interpretational' possibility**
  - Linnebo motivates this idea via the apparent possibility of changing acceptable interpretations of our language (including quantifiers) by adopting *abstraction principles*
    - c.f. Frege's famous example of introducing 'directions', by stipulating that two lines have the same direction iff they are parallel.

Interpretational possibility

Crucially for set theory, they propose that:

- someone who is currently talking in terms of one actualist hierarchy of sets can add a layer by adopting **dynamic abstraction principles** which say that (among other things)
  - for every plurality of sets *xx* in the old sense of the term, there's to be a 'set' (in the new language) which has all and only this plurality of old sets as elements.
- and this process of adding more layers can be arbitrarily (incl. transfinitely) repeated.
  - $\Diamond\phi$ is true iff you could make ϕ true via some **well-ordered sequence of acts of reconceptualization** (whether or not it would be metaphysically possible for anyone to make such a sequence of abstractions)

Dependence potentialists don't specify the intended structure of an iterative hierarchy in their paraphrases.

- Instead, they take many facts about the kind of structure of sets you can start talking in terms of to fall out of the *interpretational essence* of sethood and elementhood
  - e.g. extensionality is preserved in all relevant reinterpretations of 'set' and 'element'

This lets dependence theorists give shorter logical regimentations for set theory than minimalists can.

Minimalist paraphrase of "$(\forall x)(\exists y)(x \in y)$" (where quantification over all V_i is shorthand for quantification over all second-order objects X, f satisfying some axioms like PA_2)

$$\Box(\forall V_1)(\forall x)[x \in V_1 \rightarrow \Diamond(\exists V_2)(\exists y)(y \in V_2 \wedge V_2 \geq V_1, \wedge x \in y)]$$

Dependence paraphrase (using 'set' as a primitive):

$$\Box(\forall x)[set(x) \rightarrow \Diamond(\exists y)(set(y) \wedge x \in y)]$$

So I admit one practical advantage for the dependence theorist: shorter and cleaner looking paraphrases!

- But typesetting isn't all (and minimalists could Linnebo's notation as an abbreviation).

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Now let's turn to the worry for minimalist potentialism I want to consider

Linnebo and Studd both explain their notion of interpretational possibility partly by considering what you could get via

- some well-ordered sequence of reconceptualizations events
- of arbitarary length

So you might think...

- Don't we understand this notion of 'in principle possible well-ordered sequences of stipulation events (of arbitrary length)
  - via something like logical possibility, which applies equally (and analogously to) propositions involving any objects and relations?
- If so, dependence theorists seem to understand, and invoke something very close to a minimalist conception of **the ordinals**. For compare:
  - Linnebo's possible well-ordered sequences of reconceptualization events
  - Putnam's possible pencil points and arrows forming standard models of set theory

So if we're willing to presume understanding of minimalist potentialism about **the ordinals**

- wouldn't it be simpler and more elegant to just extend this story to minimalist potentialism about **the sets**?
- why bother with dependence theory?

Note: We can't simply argue as follows

- We should treat the sets and the ordinals similarly
- Dependence theorists 'in essence' accept minimalist potentialism about the ordinals.
- So they should also adopt minimalist potentialism about the sets.

For dependence theorists would presumably say:

- Yes we should treat talk of the ordinals and the sets the same way
- But both minimalist and dependence potentialist proposals to meet the low bar of
  - using meaningful conceptual primitives and claims
  - getting truth values for claims about the sets/ordinals right
- But overall other considerations favor giving a dependence theoretic explication for both mathematicians' talk of both sets and ordinals.

So the minimalist point above arguably needs to be paired with

- some positive reason for preferring minimalist potentialism
  - assuming both proposals meet the minimum standard above (acceptable primitives, correct truth conditions)?

I will suggests 2.5 such arguments in what follows.

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Argument from Content Conservation

One goal when choosing a potentialist explication for set theory might be to the current intuitive content and interest of set theoretic practice

- c.f. Wittgenstein's slogan that (approx) logical explication should 'rotate around the axis of our real need'[?]

Argument from Content Conservation

- In this section I'll argue minimalists can better
  - preserve the intuitive content and interest of current set theoretic practice.
  - avoid adding intuitively irrelevant commitments to its paraphrases of set theoretic claims
- even if dependence paraphrases successfully pass the low bar above (acceptable primitives, correct possible worlds truth conditions)
  - cf what's wrong with explication of heat claims with the form $M \wedge A$ where
    - M is a claim about molecular motion
    - A is some random metaphysically necessary aesthetic truth

Specifically, minimalist better fit enduringly popular intuitions about what's relevant vs. irrelevant to mathematics, of the following varieties:

- logicist
- structuralist
- barcan-marcus controversy avoiding

Logicism

Arguably logicist intuitions that **math is a part of logic (or closely related to it)** to favor explications of set theory via claims about

- logical possibility and conditional logical possibility – as per minimalist potentialism
- possibilities for *neo-carnapian language change* (interpretational possibility) – as per dependence potentialism

Structuralism

Structuralist intuitions that mathematics is the science of structure (individual natures and essences are irrelevant) arguably motivate

- minimalist potentialisms that invoke structure-preserving logical possibility $\Diamond_{...}$ rather than claims about what's logically possible for specific objects
- (and perhaps) minimalist explications over-dependence theoretic ones generally
  - since minimalist paraphrases appeal to logical possibility constraints which apply equally to all objects and relations, rather than supposed essence properties of set.

Avoiding Controversial Commitments re: Modal Logic

One might also argue dependence paraphrases of set theory add intuitively irrelevant and controversial content via

- making controversial assumptons about quantified modal logic/de re possibility required for the justifciation for (their version of) basic principles of set theory
  - Specifically: Linnebo and Studd endorse a converse Barcan-Marcus principle implying everything exists (interpretationally) necessarily.
  - maybe they can reply that 'everything exists *interpretationally* necessarily' is analytic-ish, not controversial.
    - But this makes interpretational possibility $\Diamond$ look less natural, hence a less attractive primitive.
- In contrast, (my favored form of) minimalist potentialism [2, 3] avoids taking a stance on this topic, by replacing de re possibility claims with claims about structure-preserving possibility.

So, overall, I think minimalist potentialists can claim to

- introduce less intuitively irrelevant content when explicating set theoretic claims
- and thus do a better job of ‘rotating around the axis of real need’ (or real curiosity)
    - because they adhere better to logicist and structuralist ideas about what does and doesn’t matter to mathematics

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Next, dependence theorist face a worry about whether they can answer ‘how many pure sets are there (actually)?’ in a principled way.

Imagine people who explicitly stipulate a structure for the "sets" and then change their language by making Fregean individuation stipulations a la Linnebo.

- As a neo-carnapian/(weak) QV theorist like Linnebo, I think they will succeed in talking about some actualist mathematical structure and largely speaking the truth.
- I can easily imagine a principled metasemantic story about how this determines a specific actualist reference for their set talk.
  - e.g., they count as talking about V_α where α is the number of explicit height extending stipulations they have made.

However, the dependence theorists claim that **our** concept of set has a precise actualist reading (as well as its main potentialist one) seems much more mysterious

- How can mathematicians' (or anyone's) practice bring it about that this actualist meaning for their talk is one thing rather than another?
- Linnebo and Studd say quite little about how actual practice constrains or explains actualist reference.
    - e.g., does the fact that we are inclined to accept powerset count in favor of our currently 'talking in terms of' V_α for α a limit ordinal?

Should the dependence theorist say

- Yes, because actualist reference needs to make normal mathematical talk incl. ZFC come out true (though potentialist interpretations are better/more explanatory)
- No, the powerset axiom often expresses a falsehood (on the actualist reading), but this is OK because it is true on the potentialist reading (which explains/justifies what mathematicians say)
  - given Linnebo and Studd's talk of acceptably adding a single layer (and Studd's suggestion that we may frequently unknowingly count as doing this), I'd guess they'd prefer this option

- So dependence theorists can seem committed to facts about the height of the sets which
    - don't constrain the truth or assertability of our actual set talk in any normal context (outside the philosophy room) in any obvious way
    - and hence seemingly can't be explained by our assertion practices
- e.g., We could equally say that we're currently talking about V_α, where α is
    - the number of seconds since the word 'set' was first used
    - the number of thoughts humans have had about sets or...

Accordingly they seem to face a pair of worries

- **Resurrected Arbitrariness Worry**: Do dependence theorists revive
  - the traditional platonist's *metaphysical/ontological* arbitrariness worry: why does the one true hierarchy stop there and not go farther?
  - as a *metasemantic* arbitrariness worry: why do we currently count as talking in terms of an actualist hierarchy of sets of this particular height?
- **Spinning in the Void Worry**: Isn't it odd to posit an actualist meaning for our 'set' and 'extension' talk which
  - can't be accurately deployed by most speakers on demand (c.f., competent speakers can usually semi-accurately apply a word literally on depend, even if normally they most commonly use it metaphorically)
  - (maybe) includes a precise height that makes no difference to assertability in any normal context

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Argument from Conceptual Parsimony?

Finally, you could argue minimnalist potentialsm should be favored on grounds of simplicity/ conceptual parsimony. For

- the minimalist potentialism I've skeched uses commonplace notions of FOL plus anotion of logical possibility that's independently motivated (or a natural generalization thereof to $\Diamond_{\dots}$)

But dependence theory explications of set theory require us to

- expand word meanings to include interpretational essences (not just extensions all at metaphysically possible worlds, inferential roles etc.),
- use a primative interpretational possibility opperator that can seem unnatural, in only allowing domain expansion.

So conceptual parsimony seems to favor minimalist potentialism

- However, I think this is actually debatable.

Dependence theorists can (somewhat plausibly) argue notions are independently needed to

- best state quantifier variance/neo-carnapian phil of language claims.
- best explain certain kinds of social agreement in response to stipulations introducing new kinds of objects.

Needed to best state quantifier variance?

First, dependence theorists could argue we need interpretational possibility to **best state neo-carnapian theses** about the possibility of talking in terms of more objects.
We want to state such theses

- without paradox e.g, 'there are some things I'm not now quantifying over'
- sufficiently clearly to do certain philosophical jobs
  - Answer access worries about our knowledge of which causally wimpy or weird objects exist
  - Resist a certain quine-inspired on-ramp to traditional ontology (see next slide)

Onramp to Traditional Ontology

Specifically, one might motivate traditional ontology by

- Accepting tame quantifier variance which says that
  - there are contexts where, e.g., it's true to say 'all the beers are in the fridge' though some in Australia are not
  - BUT all such cases must be understood as involving quantifier restrictions of some most natural unrestricted quantifier sense.
- Asking Quine's question 'what is there?' with regard to this favored, most natural quantifier sense
  - to get an ontological question with a right answer and traditionally expected metaphysical weight.

Using Interpretational Possibility for General NeoCarnapian Projects

I agree that we can plausibly use the interpretational possibility operator to

- coherently describe variant languages more ontologically profligate than our own
- as needed for the access worry banishing, traditional ontology deflating projects above.

But I claim we can also use the [2, 1]the conditional logical possibility operator $\Diamond_{...}$ to do the same jobs e.g., We can systematically give truth conditions for (portions of) the variant language we'd speak if we accepted ontologically inflationary postulates by saying something like the following

- For all sentences S with certain restricted vocabulary (chosen to avoid semantic vocabulary that's independently prone to generate liar paradox) and all possible worlds w,
  - "S" expresses a truth at w in the new language iff $\Box_{R_1,\ldots,R_n}$[Postulated Axioms $\rightarrow$ S] is true at w
    - where $R_1 \ldots R_n$ are antecedently understood predicates and relations whose application the postulation is not empowered to change.

So, I think we don't need an interpretational possibility operator to do the jobs above

- i.e., coherently describe variant languages more ontologically profligate than our own (not as mere quantifier restrictions) in sufficient detail to
  - block the above on-ramp to traditional metaphysics
    - (by denying that all variant quantifier senses are restrictions of a maximum metaphysically favored one)
  - reduce access worries by saying we'd have still spoken the truth if we'd accepted different coherent posits characterizing abstract objects.

However, dependence theorists might argue we need the interpretational possibility operator to state **the best version** of broadly neo-carnapian philosophy of language

- For in *Thin Objects* Linnebo advocates a form of QV which
  - takes there to be definite facts which reidentify objects before and after quantifier meaning change
- And arguably, such facts
  - can't be attractively stated using just conditional logical possibility and the trick above (since it just specifies truth-values)
  - can be attractively stated with de re interpretational possibility claims

Second,dependence theorists might argue that

- we need something like the interpretational possibility operator
  - (specifically, facts about interpretational essences)
- to explain certain coordinated responses to real world examples of (what the neo-carnapian would take to be) ontologically inflationary language change..

Here's what I have in mind

Problem of defaults

(Broadly) Neo-carnapians/weak quantifier variantists want to dispel access worries about apparent knowledge of abstract/causally wimpy/weird objects by saying that

- mathematicians and sociologists etc. introducing any suitably coherent axioms/posits
- often can and do change quantifier meanings so their postulated axioms wind up expressing truths

However in such cases, many claims about how new kinds relate to old kinds (e.g., is the square root of negative 1 identical to Julius Caesar? Is it a politician? Is it north of the Rhine?)

- Aren't logically necessitated by anything the mathematicians/sociologists introducing the new objects explicitly say
- But are promptly agreed to have a certain truthvalue by the language community at issue

So dependence theorists could argue that to explain this agreement we independently need something like interpretational essences

- i.e.,defaults for how the extensions of terms like ‘person’ and ‘is located at’ can be changed by ontologically inflationary stipulation.
    - e.g., pure mathematical objects (by default) never have location properties, and never are physical objects, people etc.
    - new kinds of composite objects whose parts are physical (by default) inherit location properties from their parts in such and such a way.

Note: I'm suggesting interpretational essence (and logical possibility) facts might reflect *defaults* – somewhat analogous to the safety mode on a computer.

- For I agree with Warren[?] we could in principle, make very radical changes, e.g. by accepting inference rules for tonk (in a suitably unreserved way) we could start speaking a language where all sentences express truths and are assertable.

Advertisement: Neocarnapian work on explaining, understanding, improving our (claimed) practices of ontologically inflationary language change might be enriched by looking at

- different programming languages' explicit rules for type introduction and inheritance in
  - comparing these languages might suggest possibilities and clarify the pragmatic costs and benefits of different ways of setting defaults.
- empirical linguistics work on shared expectations in reaction to neo-carnapian language change

But in this talk, I'm just making a tentative friendly suggestion re: indepdentently motivating the interpretational possibility operator.

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Conclusion

In this talk, I've presented 3.5 arguments for favoring minimalist potentialism.

- Given that dependence theorists accept and invoke something very like minimalist potentailist potentialisms about the ordinals, why not say the same about the sets?
- minimalist potentialism seems better position to
  - conserve intuitions about the content of current mathematics by fitting structuralist, logicist impulses (and avoiding commitments to controversial quantified modal logic claims)
  - avoid metasemantic arbitrariness worries about the height of a supposed actual height of the hierarchy of sets.

Minimalists might also have an advantage re: conceptual parsimony.

But I think this is debatable, since dependence theorists can claim we need their ideology to

- state the best form of neocarnapian theses (according to some) thin realism,
  - w/ re-indentification facts involving objects before and after ontologicaly inflationary language change (if you think this is a good thing!)
- explain socially coordinated expectations during real life ontologically inflationary language chance

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An Answer from Studd on Involuntary Quantifier Meaning Change?

In [12], Studd explains how people could unknowingly come to talk in terms of a progressively larger actualist hierarchy of sets (without their making explicit Tarskian stipulations that Linnebo imagines).

- So one might hope that this would give us some ideas about how to answer the question 'how many sets are there actually?' (and thereby the metasemantic arbitrariness challenge)
- but I'll suggest it actually just reveals another question that's hard to answer without arbitrariness.

Studd says: imagine people who

- start out speaking a language Q that 'talks in terms of' a certain hierarchy of sets
- *knowingly* attempt to split off from the main body of Q-language speakers and develop a new language E which talks in terms of extra sets.

This splinter group could achieve their ends by adopting certain principles, including the following, for reasoning from claims in the old language Q to claims in the new language E, and vice versa. .

$$Q : \mathit{things}(vv) \Rightarrow E : \mathit{thing}(\{vv\})$$

$$Q : \mathit{things}(vv), Q : v \prec vv \Rightarrow E : v \in \{vv\}$$

$$Q : \mathit{things}(vv), E : v \in \{vv\} \Rightarrow Q : v \prec vv$$

Intuitively, these schemas embody the idea that each plurality vv of objects quantified over in the old language Q is supposed to form a set in the new language E

- so the quantifiers in the new language E range over strictly more objects than quantifiers in their original Q.

Now Studd argues considers a different group of people unknowingly accepting inconsistent axioms of set theory (including the ones below) can give rise to similar kind of expansionary quantifier meaning change:

$$\mathit{things}(vv) \Rightarrow \mathit{thing}(\{vv\})$$

$$\mathit{things}(vv), v \prec vv \Rightarrow v \in \{vv\}$$

$$\mathit{things}(vv), v \in \{vv\} \Rightarrow v \prec vv$$

The above inference principles are inconsistent in a familiar Russellian way[2]

[2] They let you infer that, for any plurality of things vv, there's a set $\{vv\}$ whose elements are exactly the objects v in this plurality vv (written $v \prec vv$). But accepting the existence of this set (together with normal plural comprehension principles saying that, for any ϕ, there's a plurality vv of the objects such that ϕv) lets you derive the existence of the Russell set and hence contradiction.

Studd suggests these speakers endorsing the principles above would unwittingly undergo quantifier meaning change (to start thinking in terms of more sets).

- it's more charitable to interpret this second group of speakers as undergoing language change analogous to the switch from Q to E envisaged above, than as saying something inconsistent.

Studd puts this proposal forward (cautiously) as, "[the] basis for an idealized account of universe expansion applicable to the ordinary English speaker." Objections:

- Surely modern people aware of Russell's paradox don't have the paradoxical inference dispositions above.
- (more relevant to this talk) If I have the inconsistent inference dispositions Studd mentions and don't think about set theory for an hour, how many times is Studd's process of language change supposed to occur? It's hard to imagine a non-arbitrary positive answer to this question.

But debatably there's a fourth job in general neo-carnapian language we genuinely need interpretational possibility (not merely $\Diamond_{...}$) to do.

- In *Thin Objects* Linnebo advocates **thin realism** on which there are definite facts about identity of objects through language pre and post ontologically inflationary language change
- Such identify facts plausibly
  - can be stated with de re interpretational possibility claims
  - can't be stated using logical possibility and the trick above (since this just specifies truth-values)
- So if you accept Linnebo''s arguments for thin realism, this could independently motivate accepting (something like) quantifying into the interpretational possibility operator as meaningful

Arg 1 for thin vs. ultrathin realism

Arg1: Linnebo criticizes ultra-thin realism for allowing new languages as talking in terms of new objects in cases where the apparent predicate subject structure of expressions doesn't contribute to truthvalues for sentences in the usual systematic way.

- Response: Accepting this crit doesn't require us to accept de re interpretational possibility facts.
- Indeed all the ontologically variant languages describable via the if-thenist trick above will
  - have quantifiers and object/predicate structure of sentences which contribute to truthvalues in a familiar way.

Arg 2 for thin vs. ultrathin realism

Arg 2: If (contra thin realism) we can start taking in terms of more objects via stipulations that don't fix identity relations between objects of the new and old language, then there will be facts about reference in the new language which lack adequate explanation

- Response: I'm not sure whether such reference without explanation is a problem, if you think of language change events as suggesting ways of recarving reality
  - is this lack of explanation any worse than the lack of explanation for why 'i' names i rather than $-i$?
  - could we accept indeterminacy about these facts, as per Parsons saying it's indeterminate which sets is identical number one?

  But perhaps I'm missing something
  - e.g. do the above response require objectionable indeterminacy of reference in our mother tongue?

And overall, I want to admit stating thin realist quantifier variance facts about identity between objects in new and old language

- a reasonably promising candidate for a job we independently need an interpretational possibility operator to do.

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