Note: Importing and justifying S5

August 21, 2024

Errata:

- As per the description, statement of importing (on pg 96 of the paper back) should have a $\Diamond$ rather than a box. So it should read $(\Theta \wedge \Diamond_{\mathcal{L}}\Phi) \to \Diamond_{\mathcal{L}}(\Theta \wedge \Phi)$

Here are some answers to questions (thanks to Chris Scambler)

More explanation of how to get S5:

T: $\Box A \to A$.
Assume $\Box K$. That's $\neg\Diamond\neg A$. Suppose for contradiction $\neg A$. Then $\Diamond\neg A$ by $\Diamond I$. Contradiction.

K: $\Box(A \to B) \to (\Box A \to \Box B)$
Assume $\Box(A \to B)$ and $\Box A$. Suppose, for contradiction, that $\Diamond\neg B$. Then $\Diamond[\Box(A \to B) \wedge \neg B]$ by Importing (since $\Box(A \to B)$ is content restricted to the empty set), $\Diamond(A \to B) \wedge \neg B$ by logical closure (since $\Box(A \to B) \vdash A \to B$ by $\Box$ E [or T above]. So $\Diamond\neg A$ by logical closure (since $(A \to B) \wedge \neg B \vdash \neg A$). But this contradicts $\Box A$, i.e., $\neg\Diamond\neg A$. So we have $\neg\Diamond\neg B$ aka $\Box B$.

5: $\Diamond A \to \Box\Diamond A$.
Assume $\Diamond A$. Suppose for contradiction that $\Diamond\neg\Diamond A$. Then, since $\neg\Diamond A$ is content restricted to the empty set, we can apply $\Diamond$ elimination to get $\neg\Diamond A$. But this contradicts our assumption that $\Diamond A$. So $\neg\Diamond\neg\Diamond A$ aka $\Box A$.