QUANTIFIER VARIANCE, MATHEMATICIANS' FREEDOM AND THE REVENGE OF QUINEAN INDISPENSABILITY WORRIES

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Abstract. Invoking a form of Quantifier Variance promises to let us explain mathematicians' freedom to introduce new kinds of mathematical objects in a way that avoid some problems for standard platonist and nominalist views. In this paper I'll argue that a variant on Quine's classic indispensability argument seems to pose a problem for the quantifier variance explanation of mathematicians' freedom and then suggest three possible solutions to this problem.

1. Introduction

Invoking a form of Quantifier Variance promises to let us attractively explain mathematicians' freedom to introduce new kinds of mathematical objects. For according to Quantifier Variance, when mathematicians introduce hypotheses characterizing new types of objects, this choice can simultaneously give meaning to newly coined predicate symbols and names and change the meaning of expressions like "there is", in such a way as to ensure the truth of the relevant hypothesis[1] . Thus, for example, mathematicians' introduction of the complex numbers might change the meaning of our quantifiers so as to make the sentence "there is a number which is the square root of -1" go from expressing a falsehood to expressing a truth.

[1]See, for example, Hirsch[13], who coined the term Quantifier Variance, [22], REDACTED and the discussion about whether Carnap is best understood as advocating Quantifier Variance in [9]. However existing proponents of this explanation for mathematicians' freedom have tended to share a background metaontological antirealism, which (we will see below) is not obviously required by this explanation. To help clarify this fact, I will define Quantifier Variance to be a somewhat weaker thesis than Hirsch takes it to be (one that doesn't include a commitment to metaontological antirealism).

In this paper I will discuss a problem for the Quantifier Variance explanation of mathematicians' freedom which might be called the 'Revenge of Quinean Indispensability Argument'. Because the Quantifier Variantist accepts the literal existence of mathematical objects, the classic Quinean Indispensability problem doesn't make trouble for them. However, I will argue that a grounding-based version of classic Indispensability worries does arise for the Quantifier Variantist.

In sections 2 and 3 I will motivate and review the Quantifier Variance explanation of mathematicians' freedom. In section 4 I will develop the Revenge of the Quniean Indispensability challenge. And in section 5 I will discuss three options for answering it.

2. Motivating Quantifier Variance

Contemporary mathematical practice seems to allow mathematicians significant freedom to introduce new kinds of mathematical objects, such as complex numbers, sets and the objects and arrows of category theory. For example Julian Cole writes, "Reflecting on my experiences as a research mathematician, three things stand out. First, the frequency and intellectual ease with which I endorsed existential pure mathematical statements and referred to mathematical entities. Second, the freedom I felt I had to introduce a new mathematical theory whose variables ranged over any mathematical entities I wished, provided it served a legitimate mathematical purpose. And third, the authority I felt I had to engage in both types of activities. Most mathematicians will recognize these features of their everyday mathematical lives." [5].

Philosophers of mathematics face a challenge about how to account for this, and they have developed a number or styles of response. One style of response is (what I will call) the Quantifier Variance explanation of mathematicians' freedom.

To informally state and motivate this view, consider what it is natural to say about our knowledge of objects like holes and shadows - which do not appear to be identifiable with mereological fusions of particles or region of space. In ordinary contexts we appear to quantify over objects like holes in a road or in a piece of Swiss cheese. For example, we may say that there are three potholes in the road between one town and another, or that one piece of cheese has more holes in it than another. And if one accepts the existence of these holes, it is appealing to think of them as distinct from things like the air that occupies them or surrounding portions of the 'hole host' (e.g., the cheese or the pavement) [2].

Is there an access problem about our knowledge of holes? One might try to get such an access worry going, by arguing as follows. Our ability to visually determine how many holes there are in a road, depends on our (implicit or explicit) accuracy concerning how hole facts supervene on facts about the distribution of 'solid' matter in space[3]. For example, we must be disposed to make correct judgments about how deeply indented a road must be to count as containing a hole. But what can explain the match between our beliefs on this topic and the corresponding objective reality about when there is a hole in the road? It doesn't seem like sensory experience or scientific practice strongly motivates thinking that any particular place to draw the line is intrinsically physically/metaphysically special (even allowing for some vagueness). Thus people's apparent ability to draw the line correctly could seem to create an access problem[4].

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[2]See [15]

[3]Though see REDACTED for a puzzle about this notion of solidity,

[4]Note that the issue with this 'access problem for holes' is not supposed to be about vagueness, but about our ability to be accurate (or even close to accurate) about how hole facts supervene on indentation facts. Consider the match between *facts* of the form:

However, it's appealing to say that there isn't really any such access problem for holes, because one can give the following metasemantic explanation for human accuracy about minimum hole indentation facts and the like. If we had been inclined to say something (logically coherent but) different about when an indented object counts as 'containing a hole', then the meaning of the words "hole" and "there is" would have been different, so that our utterances would have still expressed truths. That is, we would have been speaking as slightly different language in which a slightly different collection of sentences of the form, "Whenever a solid road is indented in according to geometrical formula ϕ there is a hole in it" express true propositions[5]. Accordingly, there's no mystery or spooky Leibnizian predetermined harmony in our possession of true beliefs about things like about how steep holes must be [6].

Note that the explanation above seems to involve Quantifier Variance, in that it requires that our adopting different hole attribution practices would have caused a shift in the meaning some logical vocabulary like the existential quantifier (not just a shift in the meaning of the world 'hole'. For example, note that changing between more and less generous standards for hole existence could require the truth value of the Fregean sentence

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'When a road is missing a cylinder of material of depth 3cm and width 15cm, there is a hole in that road.'
'When a road is missing a cylinder of material of depth .01cm and width .1 cm, there is *not* a hole in that road.'
and human *beliefs* about these facts. We process visual information in way that draws the line somewhere (maybe with some vagueness), but what explains the match between where we do draw the line and the correct place to draw the line?

[5]Arguably our current language allows for contextual variation in how strict the standards for hole existence are and hence (for the reasons to be discussed below) corresponding variation in the meaning of 'there is'. So one might think of there being a shared core meaning to 'there is' (perhaps associated with the introduction and elimination rules) which combines with contextual factors to determine truth conditions for sentences involving 'there is' at each metaphysically possible worlds. For present purposes I'll simply talk about shifts in quantifier meaning, but I don't mean to prejudge this issue.

[6]Or at least there's no mystery if we bracket access worries about knowledge of logical coherence.

which says 'There are n things' using only first order logical expressions and equality[7].

Also note that the Quantifier Variance explanation above does not suggest that when we start talking in terms of holes and shadows (or switch from stricter to laxer standards for hole existence) we bring these objects into being. The existence of holes and shadows is not caused by, or grounded in, the existence of language users who talk in terms of holes and shadows, and it will be true to say "there were holes before there were people, and before I started talking in terms of them." Instead we are merely changing our language so that some sentences, e.g., "there is something [namely, a hole] in the region of the cheese plate which is not made of matter" go from expressing a false proposition in our old language to expressing a different, true, proposition in our current language[8].

3. The Quantifier Variance Explanation of Mathematicians' Freedom

3.1. **Quantifier Variance Thesis.**

With this motivation in place, I will now characterize a Quantifier Variance thesis, and discuss how we can use it to explain mathematicians' apparent freedom to introduce new mathematical structures on the basis of considerations of mere logical coherence.

By Quantifier Variance (QV) I mean the following pair of claims:

- There are a range of different meanings "there is" could have taken on, which all obey the syntactic rules for existential quantification[9].

[7]For example, the sentence that says there are two things $(\exists x)(\exists y)[\neg x = y \wedge (\forall z)(z = x \vee z = y)]$.

[8]See[8] for a vigorous development of this point which I found very helpful.

[9]By this I mean that, for each such quantifier sense there is some possible language such that all applications of the standard syntactic introduction and elimination rules for the existential quantifier within that language are truth preserving. However, that does not mean that one can form a single language containing both quantifier senses and then apply the introduction and elimination rules to prove the equivalence of these senses. See [22], among others, on this point.

- These senses need not all be mere quantifier restrictions of some fundamental maximally natural quantifier sense (if there is one)[10].

In contrast to previous formulations[11], I will not take Quantifier Variance to include the further parity claim that all the variant quantifier senses are somehow equally metaphysically joint carving. For example, it would be compatible with Quantifier Variance as I shall understand it to say that there's a maximally natural quantifier sense corresponding to what objects exist fundamentally.

Indeed, some friends of ontology have found their own reasons for accepting the above Quantifier Variance thesis. For example Sider [21] uses Quantifier Variance to capture the intuition that ordinary speakers' non-philosophical utterances like 'there's a hole in the road' can express uncontroversially true statements, despite the fact that there's a deep open question about what exists in the more fundamental sense relevant to the metaphysics room. Sider says there's a unique, maximally natural, sense of the quantifier which ontologists aim to employ. And plausibly it's a deep open question whether holes exist in this sense. But he allows that that there are also other (perhaps less metaphysically joint-carving) senses, which the

[10] That is, these variant quantifier senses need not be interpretable only only as ranging over some subset of the objects which exist in the fundamental quantifier sense, in the way that we might say the "all' in a typical utterance of "all the beers are in the fridge" restricts a more generous quantifier sense to only range only over objects in the speakers house.

[11] More traditional neo-Carnapian formulations of Quantifier Variance [13], [9] have tended to combine the *multiplicity claim* in the **Quantifier Variance** thesis (that the "∃" symbol can take on a range of different existential-quantifier-like meanings) with a *parity claim*, to the effect that all these meanings are (somehow) metaphysically on par. Thus, for example, Chalmers characterizes Quantifier Variance as (roughly) the idea that, "there are many candidate meanings for the existential quantifier (or for quantifiers that behave like the existential quantifier in different communities), with none of them being objectively preferred to the other." [3]

quantifier can take on in ordinary contexts, such that utterances of 'There is a hole in this pipe.' express a clearly true proposition[12].

3.2. **Quantifier Variance and Mathematicians' Freedom.** If we accept the above Quantifier Variance Thesis, we can explain mathematicians' freedom to introduce new kinds of apparently coherent objects along the following lines.

Quantifier Variance Explanation of Mathematicians' Freedom: When mathematicians (or scientists or sociologists) introduce axioms characterizing new types of objects, this choice can not only give meaning to newly coined predicate symbols and names, but can change/expand the meaning of expressions like "there is", in such a way as to ensure the truth of the relevant hypotheses.

Thus, for example, mathematicians' acceptance of existence assertions about complex numbers might change the meaning of our quantifiers so as to make the sentence "there is a number which is the square root of -1" go from expressing a falsehood to expressing a truth. Similarly, sociologists' acceptance of ontologically inflationary conditionals like, "Whenever there

[12]Note that saying some kinds of objects (e.g., cities, numbers) might not exist in the sense relevant to the Sider's fundamental ontology room doesn't amount to saying that these objects 'don't really exist'. It is entirely compatible with truthful assertion that these objects literally exist in the course of daily life (and while studying ethics or the metaphysics, of money and gender, or writing philosophy of mathematics papers like this one) – much as acknowledging that rabbits don't exist on the the (relatively) more natural and joint-carving quantifier sense employed by fundamental physics is compatible with saying rabbits literally exist in most ordinary contexts, including biology seminars. When outside the fundamental physics/ontology room, our position on such objects seems much more naturally expressed by saying that rabbits/holes/cities/numbers *might not be fundamental* than that they *don't really exist*.
Also note that (as discussed in REDACTED) using quantifier variance does not require one to accept that normal English employs verbally different expressions corresponding to at least two different quantifier senses (a metaphysically natural and demanding one and a laxer one), so that it might be true to say things bad-sounding things like "composite objects exist but they do not really exist" in certain contexts. With regard to any particular context we can fully agree with David Lewis that, "The several idioms of what we call 'existential' quantification are entirely synonymous and interchangeable. It does not matter whether you say 'some things are donkeys' or 'there are donkeys' or 'donkeys exist'...whether true or whether false all three statements stand or fall together." [15]

are people who... there is an ethnic group which ..." can change the meaning of our quantifiers so as to ensure that these conditionals will express truths.

3.3. **Advantages.** Giving this Quantifier Variance explanation for mathematicians' freedom promises to let us avoid problems for more familiar ways of explaining mathematicians' freedom like classic set theoretic foundationalism and nominalism.

According to classic set theoretic foundationalism (and other broadly plenitudinous platonist views), there is a very large mathematical universe, such that all (or nearly all) logically coherent hypotheses describing pure mathematical structures have an intended model somewhere within this universe. However these plenitudinous views give rise to an arbitrariness worry. For any size the total mathematical universe has, it would seem to be logically coherent to imagine a strictly larger abstract structure[13]. So it can seem arbitrary to suppose that the plenitudinous universe stops at any particular point[14].

According to nominalism, no mathematical objects literally exist and that the standards for correct mathematical utterance don't require matching facts about what mathematical objects exist. However, as Benacerraf famously argued [1], this seems to require treating quantification over mathematical objects as unmotivatedly different from quantification over cities. It seems that the nominalist must either unattractively say that mathematicians statements are literally false[15], or say that mathematical statements

[13]We can imagine this structure being formed by adding objects which behave like a layer of classes over our original mathematical universe, and then note that the result must be larger than the original universe for Cantorian reasons.

[14]One might also argue that there are epistemic problems about our knowing where it stops. As Wright and Shapiro say, any reason for thinking there are sets at all seems to motivate thinking that the sets continue up through any successor or limit stage[20]

[15]Recall Lewis saying, "I am moved to laughter at the thought of how presumptuous it would be to reject mathematics for philosophical reasons. How would you like the job of telling the mathematicians that they must change their ways, and abjure countless errors, now that philosophy has discovered that there are no classes?"[14]

have a different logical form from claims which ordinary speakers treat similarly (e.g, apparent existence claims about holes and countries).

Adopting the Quantifier Variance explanation for mathematicians' freedom promises to let us avoid both problems above. For the Quantifier Varianitist can say that the fact that our mathematical structures come to an end somewhere reflects a choice of what concepts to use, not an extra brute joint in reality. So are not committed to an extra (and perhaps unknowable) brute joint in reality about where the hierarchy of sets comes to a stop, in the way that set theoretic foundationalists are. And (unlike nominalism) the Quantifier Variance explanation also promises to let us honor Benacerraf's goal of treating apparently grammatically and inferentially similar talk of numbers and cities similarly. And it allows us to say that a single notion of existence is relevant to claims like "Evelyn is prim." and "Eleven is prime." in any given context (though, of course, future choices may further change which notion of existence one's language employs).

4. Revenge of the Quinean indispensability problem?

Despite the advantages outlined above, many questions can be raised about Quantifier Variance and the Quantifier Variance explanation of mathematicians freedom.

For example, worries have been raised about whether the Quantifier Variantist can say something attractive about the following. What would happen if mathematicians simultaneously adopted a pair of internally consistent, but incompatible, conceptions of pure mathematical structures? What would happen if mathematicans' adopted a conception of some mathematical structure which imposed undue constraints on the total size of the universe (e.g., a logically coherent collection of axioms describing a purported mathematical structure which imply that the total universe contains at most 100 things?)[16].

I won't discuss my preferred answers to these known objections here[17] , as my aim is to develop and discuss a different challenge to the Quantifier Variance explanation of mathematicians' freedom. I call this challenge the 'Revenge of the Quinean indispensability argument'.

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[16]One might also object to the Quantifier Variance thesis above as follows. If our use can change the meaning of our quantifiers in the way suggested above (allowing for a metasemantic answer to access worries about our knowledge of minimum hole indentation), then why doesn't fairy believers talk of fairies change the meaning of our quantifiers so 'there are fairies' is true?
However, I think this objection proves too much. For the same reasoning would call into question our access to color facts. For, presumably, the correct explanation of our accuracy regarding what colors qualify as red involves the following fact: if we were inclined to differentiate colors in another way then the meaning of red would have been different. (I take the acceptability of such metasemantic explanations to be fairly uncontroversial in cases where no quantifier variance is required to deploy them.) But one could just as well ask: if our use of "red" can change what predicate this word expresses (as above) then why doesn't antivaxers' use of "autism-causing" change the meaning of that predicate, so their beliefs come out true?
To my knowledge, no complete and satisfying story about how use determines meaning in either case has yet been developed. But it seems to me that essentially the same tools (appeal to a distinction between more and less definitional/would be analytic aspects of our use, appeal to more vs. less natural kinds among properties, or more vs less natural kind properties) seem available in both cases.

[17]In a nutshell, I think we can answer the first challenge by saying that mathematicians' actual (and claimed) freedom only allows a given mathematical community/context to employ any logically coherent *total collection* of conceptions of pure mathematical structures (see work in progress Tom Donaldson on this point). So a proponent of the Quantifier Variance explanation of mathematicians' freedom can say that if mathematicians simultaneously employ a pair of incompatible conceptions of mathematical structures (in some context) a) this would be an accident and b) at most one of these conceptions of mathematical structures would express a truth. And we can answer the second challenge by noting that axioms characterizing pure mathematical objects always employ quantifiers that are implicitly restricted to some collection of pure mathematical structures (see the discussion of implicit quantifier restriction in the Peano Axioms in REDACTED), so these conceptions cannot impose any restrictions on the total size of the universe.
Perhaps a fuller answer to these challenges would fit the above claims into a general metasemantic story which also yields attractive verdicts about our practice of talking in terms of objects like holes, cities, contracts etc. In REDACTED I propose a formal device (a method for describing truth conditions for languages that are more ontologically profligate than one's own) which may help with this project, and begin to sketch such a theory.

Recall that the famous Quinean indispensability argument challenges the nominalist to (on pain of hypocrisy) state their best scientific theories without quantification over mathematical objects. If one believes we ought to accept the existence of all entities indispensable to (literally) stating our best scientific theories (as Quine suggests we should[18]), then this puts a burden on the nominalist to convince us our physical theories can be stated without recourse to mathematical objects. And this task has proved notoriously difficult.

As the Quantifier Variantist acknowledges the existence of mathematical objects, it seems that they can quantify over mathematical objects in their best scientific theories without risk of hypocrisy. But, while Quantifier Variantists dodge the classic Quinean indispensability argument, something feels troubling about the idea that merely allowing for certain kinds of language change can dissolve such a difficult problem. And it's natural to suspect that a version of classic Quinean indispensability worries reappears in terms of grounding.

In particular, if one accepts Siderean realism about metaphysics, all facts should be grounded in terms of facts about fundamental objects. And one might think that accepting the Quantifier Variance explanation of mathematicians' freedom requires rejecting the idea that any mathematical structures are metaphysically fundamental. So it should be possible to ground all facts involving mathematical objects in facts that don't involve mathematical objects[18]. And this challenge (to provide a nominalistic grounding for all statements they accept) looks structurally to the nominalist's classic indispensability challenge, to formalize all statements they accept without quantification over mathematical objects. And many people are pessimistic

[18]See [7] for discussion of how we might grounds facts of the form $n = m$ and 'The number of Fs = the number of Gs' in either facts about second order relations between concrete objects or in nothing.

about the prospects for straightforwardly answering the indispensability challenge, on historical grounds. So one might take nominalists' history of failure to answer the indispensability challenge to motivate pessimism about the possibility of Quantifier Variantists answering their analogous grounding problem.

Admittedly, it's not too hard for Quantifier Variantists to answer the grounding challenge noted above as it applies to pure mathematical statements, and certain kinds of simple applied mathematical challenges. For they can simply repurpose nominalists' existing logical regimentations of these statements as stories about grounding[11][19].

However, it's much less clear that one can adequately ground statements of applied mathematics (especially ones that make complex claims involving magnitudes like length and charge or probabilities). One might think that whatever blocks classic nominalists from systematically (nominalistically) paraphrasing contemporary physical theories involving objective probability (and the like), will also block Quantifier Variance Nominalist from providing an adequate grounding for such claims.

Thus we can consider a 'Revenge of Quinean Indispensability' which proceeds as follows.

(1) All facts are grounded in facts involving grounding fundamental objects and relations, plus some logical and perhaps modal vocabulary. (So, for example, demands for grounding are legitimate and there are no infinite descending chains of grounding).

[19]Hellman systematically pairs each such natural language sentence S with a nominalistic statement ϕ, which is supposed to capture the true logical form of S. The Quantifier Variantist can accept the surface logical form of S but instead take Hellman's paraphrase strategy to show how the truth of S (and the existence of any mathematical objects it quantifiers over) can be seem as systematically grounded in the nominalistic fact ϕ. And [2] suggests a way of conceptually simplifying these paraphrases.

(2) If the Quantifier Variance explanation of mathematicians' freedom (QVEMF) is true, then there is nothing metaphysically special about the conceptions of pure mathematical structures we currently employ; all logically coherent mathematical posits are 'on par'.

(3) So by (2) if QVEMF then no pure mathematical objects are grounding fundamental. (Perhaps the existence of all pure mathematical objects is grounded in facts about logical possibility/coherence of our conceptions of these structures, and/or the logical necessity that any objects satisfying this conception must have some further features).

(4) Grounding fundamental relations can only have grounding fundamental objects as relata.

(5) So, if QVEMF, then no grounding fundamental relations take pure mathematical objects as relata.

(6) So if QVEMF all facts (including ones we would normally state by apparently quantifying over mathematical objects) are ultimately grounded in nominalistically acceptable stuff like relations between non-mathematical objects, modal facts about logical possibility and the like.

(7) Any adequate story grounding applied mathematical facts in nominalistically acceptable stuff could (easily) be transformed into a nominalist paraphrase for these facts.

(8) So nominalists' historic failure to provide an adequate nominalistic paraphrase for certain scientific statements (as per the classic indispensability challenge) provides strong reason to think no adequate nominalistic grounding for these statements is possible.

(9) So nominalists' historic failure to solve the classic Quinean indispensability problem provides strong reason to think the analogous

grounding problem cannot be solved – and thus that QVEMF is false.

There are a number of possible strategies for resisting this argument. For example, readers with nominalist sympathies will probably reject (8), denying that philosophical history provides strong reason for pessimism about either nominalistic paraphrase or nominalistic grounding.

In what follows I will only discuss three options (which I think are apt to be under-appreciated). These are:

- Arguing (contra 7) that the grounding challenge might be significantly easier to solve than the classic indispensability challenge (as traditionally understood),
- Showing that (contra 2 and 3) QVEMF is compatible with some pure mathematical structures being grounding fundamental and thereby metaphysically special.
- Rejecting the demand for grounding (contra 1).

However I don't mean to suggest that they are the only plausible options.

5. Responses

5.1. **Grounding Easier than Paraphrase.** Let us begin with the first strategy mentioned above: resisting the inference from a history of failure to nominalistically *paraphrase* certain scientific theories to pessimism about nominalistically *grounding* these theories. I will discuss two (compatible) ways of motivating this idea.

First, one might argue that nominalistically grounding facts about applied mathematics is easier than nominalistically paraphrasing such facts in a few important ways.

For one thing, giving a nominalisitic paraphrase of a scientific theory ϕ mentioning mathematical objects ϕ with a *single*/finite equivalent theory which does not. However it seems independently attractive to say that we can ground a single fact in infinitely many other facts[20]. For example, it seems independently appealing to say that the truth of'there is a cat' could be partially/totally grounded in infinitely many facts about the existence of particular cats (Bess, Mrs. Whiskers etc.) if there were infinitely many cats. And it seems appealing to say that a total grounding for the fact that there are infinitely many cats (in worlds where there are) could involve the fact that Bess is a cat, that Mrs. Whiskers is a cat etc.

For another thing, it is widely thought that a systematic logical regimentation of a person's natural language statements cannot employ infinitely many different atomic predicates and relations, because a language with infinitely many atomic predicates would be unlearnable[6]. But once we replace the nominalists' task of describing the true logical structure of our scientific beliefs with the task of saying what facts about fundamentalia ground the truth of these beliefs, (I think) this argument from learnability no longer applies. So it's not clear that grounding facts couldn't involve infinitely many atomic predicates.

For there is (prima facie) no reason to assume that human beings must be able to learn distinct names for all the atomic properties which would be used in a maximally metaphysically joint carving language. For example, maybe there are infinitely many different metaphysically fundamental properties corresponding to all possible lengths (or relations corresponding to all possible length ratios). And in *Mathematics Without Numbers* Hellman actually provides an example of how one might think about physical magnitude facts as grounded in (infinitary) facts about the application of an

[20]Admittedly this suggestion has some structural similarities to Melia's defense of nominalism in [17]. See the appendix for a note about how an influential criticism of Melia doesn't apply to this proposal.

infinite number of nominalistically acceptable physical magnitude properties and relations (i.e., ones that don't take mathematical objects as relata) [21][22].

Second, one can argue that (even if nominalistic paraphrase is not intrinsically easier than nominalistic grounding) the nominalistic grounding problem is easier to solve than *the specific version* of the indispensability problem which so much manpower has already gone into trying to solve. Specifically, one might suspect that many nominalists trying to provide paraphrases for scientific theories were working under significant constraints which the proponent of QVEMF need not accept. And if this is so, it provides reason

[21] See section 3.4 of [11] and note Hellman's explicit comment that complete success in this project would not suggest that we could remove quantification over mathematical objects from our best scientific theories.

[22] Admittedly this style of response to the Revenge of Quineian indispensability challenge is somewhat (structurally) similar to a Melia's defense of nominalism against Quine's classic indispensability challenge in [17].

Melia motivates the idea that "we should not always believe in the entities our best physical theory quantifies over" by considering examples like 'the average star has 2.4 planets' where quantifying over mathematical objects is helpful because it lets us express our agnosticism between an infinite disjunction of different nominalistically statable scenarios (there are 5 stars with a total of 12 planets or 10 stars with a total of 24 stars or 15 stars with a total of) and physical magnitude facts, such as facts about lengths, which he suggests might be best thought of as involving nominalistic objects like paths through space which are related by an infinite (and therefore not human learnable) collection of atomic two place relations like 'path q is π times longer than path q' and 'path q is 3.1 times longer than path q'. In both cases it seems that a nominalistic language might adequately describe some aspect of physical reality (how many planets vs. stars there are, or what the length ratios are between different physical objects), despite the fact that we cannot satisfy Quine's criterion by giving this nominalistic description.

Now an influential line of criticism maintains that if we accept Melia's proposal — or in any other way drop the requirement that someone engaged in ontology state their best total theory of the world without quantifying over any objects they want to deny exist (in the sense relevant to the ontology room) – then we get a scenario where 'anything goes' as regard to ontology. That is, we loose any concrete grip we may hope to have had on how to settle ontological questions – and thereby perhaps any grip on what questions of traditional ontology mean. I'm not sure whether this criticism ultimately works against classic nominalists like Melia. For the the inference from, 'if P then we don't have a coherent and fruitful grip on the project of philosophical ontology' to 'therefore $\neg$P' can seem like a case of unjustified wishful thinking.

But even if this argument cut ice against a nominalist like Melia, we should note that the quantifier variantist who rejects mathematical fundamentalia and demands for finitary grounding has special tools for answering it which the nominalist does not. For, philosophers are already independently working on a theory of grounding and formal constraints on when one thing can be said to be grounded in another, and the Quantifier Variantist can say that this prevents it from being the case that 'anything goes' with respect to grounding.

to resist the inference from nominalists' failure to find a paraphrase they consider acceptable to the impossibility of providing nominalistic grounding (in the sense relevant to QVEMF).

For one thing note that many philosophers are inclined to accept nominalism about mathematical objects because they have a general resistance to accepting 'strange', (i.e., necessary, abstract and/or non-material) objects. Thus when providing a nominalistic paraphrase in response to Quine's challenge, they will want to avoid quantifying over these objects as well.

In contrast, the QVEMF theorist's motivation for saying that no pure mathematical objects are fundamental doesn't commit them to a general rejection of strange objects (in the sense above) as fundamentalia. They are motivated by a very specific features of mathematical practice (namely, mathematicians' taking themselves to be free to adopt arbitrary logically coherent conceptions of pure mathematical structures) to say that languages talking in terms of different pure mathematical structures are metaphysically on par, and no pure mathematical structures can be grounding fundamental.

But no analogous freedom is claimed by physicists in their practice of talking in terms of strange objects such as events, electromagnetic fields, the wave function in Quantum Mechanics (if we construe this as something physically real), or the 'space' in which the wave function lives. Thus the proponent of QVEMF has no reason to deny that some of these objects could be grounding fundamental. Hence they are free to use a much a wider range of non-mathematical objects in their grounding story than most traditional nominalists would be happy using in their paraphrase story[23].

[23]Relatedly, one might argue that philosophers giving a traditional 'hard-road' response to the Quinean Indispensiblity challenge (by providing a paraphrase for their best scientific theories) are likely to try to write paraphrases using notions Quine would accept. Thus, they are likely to use only first order logical quantification, and to avoid use of modal vocabulary (like a notion of logical possibility/coherence. In contrast, when the QVMF theorist attempts to explain how applied mathematical facts could be grounded in facts about relations between non-mathematical objects plus facts about logical possibility, they

To support the idea that allowing such such physically weird metaphysical fundamentalia can make the nominalists' task easier, note that avoiding quantification over events in probability statements is often cited as the largest sticking point for nominalist trying to answer indispensability worries[16].

5.2. **Agnostic Platonism.** Next, suppose we grant that the history of debate over Quine's indispensability argument suggests some mathematical objects are among the fundamentalia. Proponents of the QVEMF can still resist the Revenge of Quinean Indispensability argument above by rejecting the inference that all coherent conceptions of mathematical objects must be metaphysically on par, and thence the argument that no mathematical objects can be grounding fundamental. Saying that some mathematical structures are metaphysically fundamental might seem to raise access worries (over and above the access worries about access to facts about logical coherence which the QVEMF theorist already faces[24]). However, it need not do so.

In this section I will try to outline a view that attractively combines the quantifier variance explanation of mathematicians' freedom with the idea that (plausibly) some mathematical objects are metaphysically special and grounding fundamental. I will call this view agnostic platonism. The key idea is to say that, although the fundamentalia plausibly do include *some* mathematical objects, we don't know (and perhaps can never know) *which*[25].

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are free to use modal notions like logical coherence/possibility. For a menu of options for how such modal vocabulary can be useful see chapter 3 of [12].

[24]See REDACTED for an argument that these access worries about logical coherence are solvable.

[25]Perhaps grounding considerations motivate thinking that *some* collection of mathematical objects which are sufficiently plentiful and richly structured to do certain work in applied mathematics exist fundamentally. But, as has often been remarked (For example, see [4]) indispensability considerations don't seem to justify belief in any particular mathematical structure as different mathematical structures seem capable of doing the same work in regimenting/grounding our physical theories.

The agnostic platonist allows that some mathematical objects may well be fundamental and play a necessary role in grounding facts. But they avoid classic access worries (about mathematicians' knowledge) by saying that getting math right doesn't require guessing which mathematical structures are among the fundamentalia. One might think that a similar access worry arises with regard to metaphysicians' knowledge of which mathematical structures are grounding fundamental. However, we can answer this access worry by noting that there's no access to account for. Metaphysicians don't even *appear* to know very much about which mathematical structures are metaphysically fundamental.

In slogan form, someone who accepts agnostic platonism would say: maybe some mathematical structures are metaphysically special, but mathematicians don't care which ones those are, and they don't need to care in order to reliably form true mathematical beliefs and satisfy the epistemic aims of the project of pure mathematics!

Note that this idea (that reliably speaking the truth in mathematical ordinary language doesn't require knowing the right answer to corresponding metaphysical questions about fundamental ontology) mirrors what it is natural to say about our knowledge of holes. It may turn out to be the case that some particular hole-like notion (maybe the topological notion of holes) will be used in physics, but construction workers can draw the line where they want with regard to hole boundaries and reliably speak the truth without having to take any such stance regarding fundamental metaphysics.

Now a reader sympathetic to classic set theoretic foundationalism might object: how I can endorse the arbitrariness based criticism of set theoretic foundationalism in section 3.3 while advocating Agnostic Platonism about mathematical fundamentalia without hypocrisy? For one might worry that dividing up mathematical objects into those with fundamental existence vs. those without is just as arbitrary as saying that the hierarchy of sets just happens to stop at a certain point. And isn't being committed to arbitrariness in which mathematical objects are fundamental just as bad as being committed to arbitrariness in size of the total mathematical universe?

Even if this charge of hypocrisy were correct, I think the Quantifier Variantist view advocated above would still be an improvement on classic set theoretic foundationalism. For the arbitrary joint posited by the agnostic platonist doesn't constrain acceptable mathematical practice, whereas that posited by the set theoretic foundationalist/plenitudinous platonist does. The agnostic platonist need not admit any limits on which logically coherent pure mathematical structures mathematicians could choose to talk in terms of. For they don't think mathematicians can only introduce or study structures which are grounding fundamental. In contrast, the plentudinous platonist takes there to be a total mathematical universe (e.g., the hierarchy of sets), and holds that any conception of a pure mathematical structure mathematicians could legitimately adopt must have an intended model within it.

However, I will now sketch a more aggressive defense against this charge of hypocrisy. If the other assumptions needed for the Revenge of Quinean Indispensability Argument hold (i.e., we need to provide grouding, and mathematical objects appear indispensible to that task) then it seems that everyone, not just the agnostic platonist, must admit that certain mathematical structures are special in that they play a role in grounding non-mathematical facts about the world (e.g., maybe length reflects a fundamental facet of reality and length facts require grounding in the real numbers).

So agnostic platonism still has the advantage that it only requires us to posit that one special joint in the space of coherent conceptions of mathematical structures (specifying which particular mathematical structures play a role in grounding and/or constituting particular applied mathematical facts, e.g., facts about events and probability, or lengths) where the classic set theoretic foundationalist is committed to two positing two joints in reality (this joint, plus the joint determining where the hierarchy of sets happens to stop). That is, both philosophers will be committed to facts like 'the pure mathematical objects which play roles in grounding physical facts are exactly the real numbers and three layers of sets over them'. But the set theoretic foundationalists will also be committed to a fact like 'the hierarchy of sets just happens to stop at X point' (where that point is usually taken to occur way above the point where all sets used in physical theories exists/what is needed to contain models for all mathematical structures used in physics).

Moreover, it seems more plausible that facts about the fundamental laws of physics might provide a, as yet undiscovered, principled division between those mathematical objects which play a role in grounding applied mathematical facts and those which don't, than it does that some choice of a height for the hierarchy of sets will turn out to be principled[26]

Thus, to summarize, I think the (admittedly prima facie strange) idea of saying that, although mathematicians can introduce any pure mathematical structure they like, some pure mathematical structures are metaphysically

[26]Indeed, one might argue as follows. Applied mathematics hasn't seemed to motivate a unique choice of which mathematical structures exist, because (from a traditional platonist point of view) the total collection of mathematical objects must do two jobs. It must make sense of applied mathematics and everything we could study in pure mathematics. Given this goal, it has seemed natural to consider both, e.g., both a free standing real number structure and a copy of the real numbers within various larger structures, like the hierarchy of sets (containing objects for pure mathematical study), as candidates for mathematical reference within our best physical theories. [There's no uniquely natural choice of a collection of mathematical objects which does both jobs.]
However the agnostic platonist does not expect fundamental mathematical objects to do both these jobs. (As noted above) they can take the truth of existence claims about pure mathematical objects to be grounded in something like facts about logical possibility. Thus it seems more plausible that whatever aspects of our best physical theories make appeal to some fundamental mathematical objects indispensable (if such there are) should suggest a unique most natural collection of mathematical structures to take to be grounding fundamental.

special and instantiated by objects which are grounding fundamental is more appealing than it first seems.

5.3. **A More Carnapian Approach to Fundamentalia.** Finally, one could reject the idea of grounding, or accept something structurally very like grounding but reject the idea that the small collection of objects we choose for it must be metaphysically special.

Adopting this strategy might seem to require giving up metaphysical parsimony intuitions. For the notion of grounding seems to provide an attractive way of reconciling a complex universe and variegated language with ultimate metaphysical parsimony. On this picture there's a single small collection of maximally joint caving concepts, such that facts about them grounds everything (i.e., grounds the truth of all facts expressible in other languages using less natural concepts, and accounts for the joint-carvingness of all reference magnetic concepts employed in these languages)[27].

However, even if we reject the concept of grounding (and the idea of a small collection of metaphysically special concepts and types of objects) we can still honor parsimony intuitions. We just need to cash them out differently. For example, we might use Augustin Rayo's symmetric 'nothing but' relation[19] or perhaps something broadly like Wright and Hale's idea of two facts being conceptual recarvings of the same content[28]. We could then say that reality is 'simple' in the sense that all facts expressible in our language (and maybe some specified range of other languages) bear this 'nothing but' relation to facts in some simple 'basis language'/'basis facts'. And we can provide the same kind of systematic paraphrases/groundings considered above (where we systematically pair sentences in a apparently

[27]And older ways of honoring metaphysical parsimony intuitions, like belief in a sparse ontology such that very few (kinds of) objects exist in any sense, and all true statements in any language are best understood as 'really' only asserting the existence of these objects seem even less friendly to the Quantifier Variantist.

[28]Though see Wright and Hale's vigorous rejection of Quantifier Variance in [10].

richer language with ones in an apparently narrower one), in support of the claim that some choice of a basis language/facts/ideology is adequate.

If we cash out metaphysical parsimony intuitions in terms of something like Rayo's symmetric 'nothing but' relation (in contrast with the grounding relation), we need not suppose that there is some unique most fundamental choice of basis language. Nor need we suppose the particular conceptions of pure mathematical objects employed by some adequate basis language are somehow metaphysically special. Rather, we can say that any sufficiently expressive pure mathematical language can be combined with some small collection non-pure-mathematical vocabulary to form an adequate basis language.

Thus, I claim, we can honor metaphysical parsimony intuitions (and even keep something very similar to the traditional project of metaphysical analysis in business!) without accepting the notions of grounding and a unique choice of fundamentalia which give rise to the Revenge of Quinean indispensability worry above.

So, for example, there might be a number of small languages $L_1 ... L_n$ (corresponding to different choices mathematicians could have made about which conceptions of pure mathematical structures to adopt), which differ in which mathematical objects they talk in terms of but are all equally adequate choices for a basis language. And facts statable in any one of them L_i (plus the fact that its concepts are joint carving) could be used to (in some relevant sense) account for the truth of statements and joint carvingness of concepts employed in all others.

6. Conclusion

In this paper I reviewed some appeals of using Quantifier Variance to explain mathematicians' freedom to introduce new pure mathematical structures for study. I noted that (although its advocates have traditionally been metaontological antirealists) this explanation for mathematicians' freedom is *prima facie* compatible with metaontological realism.

I then developed a 'Revenge of Quinean Indispensability' problem for the quantifier variance explanation of mathematicians' freedom, which arises when we ask whether any mathematical objects are grounding fundamental. The existence of this problem might seem to show that the Quantifier Variance explanation of mathematicians' freedom is ultimately off limits to metaontological realists (who are more likely to take questions about grounding seriously). However, I argued that a number of promising routes are available for solving this problem, including some which are compatible with metaontological realism.

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