QUANTIFIER VARIANCE, MATHEMATICIANS' FREEDOM AND THE REVENGE OF QUINEAN INDISPENSABILITY WORRIES

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Abstract. Invoking a form of Quantifier Variance promises to let us explain mathematicians' freedom to introduce new kinds of mathematical objects in a way that avoids some problems for standard platonist and nominalist views. In this paper I'll note that, despite traditional associations between quantifier variance and Carnapian rejection of metaphysics, Siderian realists about metaphysics can naturally be quantifier variantists. Unfortunately a variant on the Quinean indispensability argument concerning grounding seems to pose a problem for philosophers who accept this hybrid. However I will charitably reconstruct this problem and then argue for optimism about solving it.

1. Introduction

Invoking a form of Quantifier Variance promises to let us attractively explain mathematicians' freedom to introduce new kinds of mathematical objects. Quantifier Variance allows one to say that when mathematicians introduce hypotheses characterizing new types of objects, this choice can simultaneously give meaning to newly coined predicate symbols and names and *change* the meaning of expressions like "there is", in such a way as to ensure the truth of the relevant hypotheses. Thus, for example, mathematicians' introduction of the complex numbers might change the meaning of our quantifiers so as to make the sentence "There is a number which is the square root of -1." go from expressing a falsehood to expressing a truth.

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In this paper I will discuss a problem for the above Quantifier Variance explanation of mathematicians freedom (QVEMF) which might be called the 'Revenge of Quinean Indispensability Argument'. Because the Quantifier Variantist accepts the literal existence of mathematical objects, the classic Quinean Indispensability problem doesn't make trouble for them. However, I will note that a grounding-based version of classic Indispensability worries does threaten the Quantifier Variantist. I will chartiably reconstruct this worry and then propose some ways to answer it.

In §2 I'll review QVEMF and how it promises to improve on traditional platonist and nominalist explanations for mathematicians' freedom. I will argue that, despite the fact that it has hitherto been associated with metametaphysical antirealism[1], QVEMF is equally available to (Sideran) realists about metaphysics. In §3 I'll develop the Revenge of Quinean indispensability argument mentioned above. Then in §4 and §5 I'll propose two ways of responding to this problem and argue for optimism about solving it. Finally, in §6, I'll discuss how a version of the Revenge of Quinean indispensability worry above makes trouble for easy road nominalists (i.e., nominalists who reject Quinean demands to literally state their best scientific theories[2]).

2. The Quantifier Variance Explanation of Mathematicians' Freedom

Contemporary mathematical practice seems to allow mathematicians significant freedom to introduce new kinds of mathematical objects, such as complex numbers, sets and the objects and arrows of category theory. For example Julian Cole writes, "Reflecting on my experiences as a research

[1]See, for example, Hirsch[18], who coined the term 'Quantifier Variance', [29], REDACTED and the discussion about whether Carnap is best understood as advocating Quantifier Variance in [13].

[2]I take this terminology from [10]. For a very developed instance of Easy Road Nominalism see [1].

mathematician, three things stand out. First, the frequency and intellectual ease with which I endorsed existential pure mathematical statements and referred to mathematical entities. Second, the freedom I felt I had to introduce a new mathematical theory whose variables ranged over any mathematical entities I wished, provided it served a legitimate mathematical purpose. And third, the authority I felt I had to engage in both types of activities. Most mathematicians will recognize these features of their everyday mathematical lives." [9].

Philosophers of mathematics face a challenge about how to account for this, and they have developed a number of styles of response. One style of response is (what I will call) the Quantifier Variance explanation of mathematicians' freedom (QVEMF) because it draws on the following Weak Quantifier Variance Thesis.

(Weak) Quantifier Variance Thesis:

- There are a range of different meanings "there is" could have taken on, which all obey the syntactic rules for existential quantification[3].
- These senses need not all be mere quantifier restrictions of some fundamental maximally natural quantifier sense (if there is one)[4].

I call the above claim the *Weak* Quantifier Variance thesis because it doesn't include a further 'parity' claim (that none of these variant quantifier senses is somehow metaphysically special) which is generally included in definitions

[3]By this I mean that, for each such quantifier sense there is some possible language such that all applications of the standard syntactic introduction and elimination rules for the existential quantifier within that language are truth preserving. However, that does not mean that one can form a single language containing both quantifier senses and then apply the introduction and elimination rules to prove the equivalence of these senses. See [29], among others, on this point.

[4]That is, these variant quantifier senses need not be interpretable only as ranging over some subset of the objects which exist in the fundamental quantifier sense, in the way that we might say the "all' in a typical utterance of "all the beers are in the fridge" restricts a more generous quantifier sense to only range only over objects in the speakers house.

of Quantifier Variance[5]. So, for example, it would be compatible with Weak Quantifier Variance to say that there's a maximally natural quantifier sense corresponding to what objects exist fundamentally.

And indeed some friends of traditional metaphysics have found their own reasons for accepting the above Weak Quantifier Variance thesis. For example Sider [26] uses Weak Quantifier Variance to capture the intuition that ordinary speakers' non-philosophical utterances like, 'There's a hole in the road.' can express uncontroversially true statements, despite the fact that there's a deep open question about whether holes exists in the more fundamental sense relevant to the metaphysics room. Sider says there's a unique, maximally natural, sense of the quantifier which ontologists aim to study/employ[6]. And plausibly it's a deep open question whether holes exist in this sense. But he allows that that there are also other (perhaps less metaphysically joint-carving) senses, which the quantifier can take on in ordinary contexts, on which utterances of 'There is a hole in the road.' clearly can express a true proposition[7].

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[5]See, for example, [18], [13] and Chalmers' characterization of Quantifier Variance as (roughly) the idea that, "there are many candidate meanings for the existential quantifier (or for quantifiers that behave like the existential quantifier in different communities), with none of them being objectively preferred to the other." [7]

[6]See the argument that (even from Sider's point of view) we don't *actually speak* a language with Sider's maximally joint carving quantifier sense in most philosophical contetexts (including discussions of metaphysics and ontology).

[7]Note that saying some kinds of objects (e.g., cities, numbers) might not exist in the sense relevant to the Sider's fundamental ontology room doesn't amount to saying that these objects 'don't really exist'. It is entirely compatible with truthful assertion that these objects literally exist in the course of daily life (and while studying ethics or the metaphysics, of money and gender, or writing philosophy of mathematics papers like this one) – much as acknowledging that rabbits don't exist on the the (relatively) more natural and joint-carving quantifier sense employed by fundamental physics is compatible with saying rabbits literally exist in most ordinary contexts, including biology seminars. When outside the fundamental physics/ontology room, our position on such objects seems much more naturally expressed by saying that rabbits/holes/cities/numbers *might not be fundamental* than that they *don't really exist.*

Also note that (as discussed in REDACTED) using quantifier variance does not require one to accept that normal English employs verbally different expressions corresponding to at least two different quantifier senses (a metaphysically natural and demanding one

If we accept the above Weak Quantifier Variance Thesis, we can explain mathematicians' freedom to introduce new kinds of apparently coherent objects along the following lines.

Quantifier Variance Explanation of Mathematicians' Freedom: When mathematicians (or scientists or sociologists) introduce axioms characterizing new types of objects, this choice can not only give meaning to newly coined predicate symbols and names, but can change/expand the meaning of expressions like "there is", in such a way as to ensure the truth of the relevant hypotheses.

Thus, for example, mathematicians' acceptance of existence assertions about complex numbers might change the meaning of our quantifiers so as to make the sentence, "There is a number which is the square root of -1." go from expressing a falsehood to expressing a truth. Similarly, sociologists' acceptance of ontologically inflationary conditionals like, "Whenever there are people who... there is a country which ..." can change the meaning of their quantifiers so as to ensure that these conditionals will express truths.

Hitherto, I take it, versions of QVEMF have largely been developed by philosophers who combine acceptance of the Weak Quantifier Variance thesis above with some strong anti-metaphysical claim (such as the parity principle above) or project [8]. However, I'm suggesting that more metaphysically realist philosophers could also adopt QVEMF (backed by the Weak Quantifier Variance Thesis above) and should consider doing so.

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and a laxer one), so that it might be true to say things bad-sounding things like "composite objects exist but they do not really exist" in certain contexts. With regard to any particular context we can fully agree with David Lewis that, "The several idioms of what we call 'existential' quantification are entirely synonymous and interchangeable. It does not matter whether you say 'some things are donkeys' or 'there are donkeys' or 'donkeys exist'...whether true or whether false all three statements stand or fall together." [20]

[8] Here I have in mind [25] and [28] as well as [18].

Giving this Quantifier Variance explanation for mathematicians' freedom promises to let us avoid some major problems for more familiar ways of explaining mathematicians' freedom like classic set theoretic foundationalism and nominalism as follows.

Classic set theoretic foundationalism (and other broadly plenitudinous platonist views[9]), posits a very large mathematical universe, such that all (or nearly all) logically coherent hypotheses describing pure mathematical structures have an intended model somewhere within this universe. And this gives rise to an arbitrariness worry as, for any size the total mathematical universe has, it would seem to be logically coherent to imagine a strictly larger abstract structure[10]. So it can seem arbitrary to suppose that the plenitudinous universe stops at any particular point.

Adopting QVEMF lets us avoid this problem by saying that the fact that our mathematical structures come to an end somewhere reflects a choice of what concepts to use, not an extra brute joint in reality. Thus proponents of QVEMF are not committed to an extra (and perhaps unknowable) brute joint in reality about where the hierarchy of sets comes to a stop, in the way that set theoretic foundationalists are.

Adopting QVEMF lets us avoid a problem for nominalism by honoring Benacerraf's thought that we should treat apparently grammatically and inferentially similar talk of numbers and cities similarly [3][11]. For example, it allows us to say that a single notion of existence is relevant to claims like

[9]See, for example [2]

[10]We can imagine this structure being formed by adding objects which behave like a layer of classes over our original mathematical universe, and then note that the result must be larger than the original universe for Cantorian reasons.

[11]It seems that the nominalist must either unattractively say that mathematicians statements are literally false (Recall Lewis saying, "I am moved to laughter at the thought of how presumptuous it would be to reject mathematics for philosophical reasons. How would you like the job of telling the mathematicians that they must change their ways, and abjure countless errors, now that philosophy has discovered that there are no classes?"[19]), or say that mathematical statements have a different logical form from claims which ordinary speakers treat similarly (e.g, apparent existence claims about holes and countries).

"Evelyn is prim." and "Eleven is prime." in any given context (though, of course, future choices may further change which notion of existence one's language employs)[12].

Even more importantly, QVEMF promises to let us avoid what's often considered the most serious problem for nominalism: the Quinean indispensability argument[13]. Recall that the Quinean indispensability argument challenges the nominalist to (on pain of hypocrisy) state their best scientific theories without quantification over mathematical objects. If one believes we ought to accept the existence of all entities indispensable to (literally) stating our best scientific theories (as Quine suggests we should[24]), then this puts a burden on the nominalist to convince us our physical theories can be stated without recourse to mathematical objects. But this task has proved notoriously difficult.

As the proponent of QVEMF acknowledges the literal existence (not to say fundamentality) of mathematical objects, it seems that they can quantify over mathematical objects in their best scientific theories without risk of hypocrisy. But, while QVEMF dodges the classic Quinean indispensability argument, something feels troubling about the idea that merely allowing for certain kinds of language change (without adopting the further controversial neo-carnapian metametaphysics fans of QVEMF have hitherto favored) can dissolve such a difficult problem. As described above QVEMF seems to combine the best features of traditional platonism (avoiding indispensability

[12]Despite these advantages, many questions have been raised about Quantifier Variance and the Quantifier Variance explanation of mathematicians' freedom. For example, worries have been raised about whether the Quantifier Variantist can say something attractive about the following. What would happen if mathematicians simultaneously adopted a pair of internally consistent, but incompatible, conceptions of pure mathematical structures? What would happen if mathematicians' adopted a conception of some mathematical structure which imposed undue constraints on the total size of the universe (e.g., a logically coherent collection of axioms describing a purported mathematical structure which imply that the total universe contains at most 100 things?). I articulate some of my preferred answers to these standard worries see REDACTED.

[13]See, for example, Field's remarks at the beginning of [14]

worries, treating mathematical objects on par with other objects) with the best features of nominalism (avoiding the 'arbitrary stopping point' worries for traditional platonism noted above, and any extra access worries it brings with it).

I will try to charitably reconstruct this worry in terms of the metametaphysical realist framework Sider develops in [27] and then answer it below.

3. Revenge of the Quinean indispensability problem?

3.1. **Basic Sideran framework.** The basic Sideran framework I'll be using has three elements.

First we have a concept of **fundamentality**, which Sider identifies with joint carvingness (in the sense in which the predicate 'is an electron' is intuitively more joint carving than the notion 'is an electron or a cow'). Importantly this question of joint-carvingness is not just supposed to apply to predicates but also to all other elements of our ideology, including the variant existential and universal quantifier meanings invoked in section 2 above. Notions can be more or less fundamental, and a notion qualifies as fundamental simpliciter if it is maximally fundamental. As noted above Sider takes there to be a single maximally fundamental existential quantifier sense. And fundamental objects are objects that exist in this unique maximally fudnamental quantifier sense.

Second, Sider endorses the following principles which connect above idea of fundamentality qua maximal joint-carvingness to expectations about some truths grounding/explaining all other truths.

- "Completeness: Every nonfundamental truth holds in virtue of some fundamental truth."
- "Purity: Fundamental truths involve only fundamental notions."

Third Sider ultimately cashes out the 'in virtue of' notion above in terms of the existence of a metaphysical semantics which accounts for language users' behavior by systematically tying their claims/utterances to claims involving only fundamental (i.e., maximally joint carving) notions[14]. Sider's examples of such a metaphysical semantics often have the form of a truth theory 'Sentence S of L is true in L iff ϕ' (where ϕ is a sentence involving only fundamentalia). But he writes "Metaphysical semantics are not required by definition to take any particular form. They must presumably be compositional in some sense (since they must be explanatory and hence cast in reasonably joint-carving terms, and must contend with infinitely many sentences). But this still allows considerable variation."[27]

I will employ this basic framework from Sider's *Writing the Book of the World* in what follows, but note that I differ from him in understanding platonism to mean the existence of mathematical objects (in our current quantifier sense) not our most fundamental quantifier sense[15]. However, this is a mere terminological difference and nothing turns on it.

3.2. **The Revenge Worry.** Now let us return to the feeling that something about the picture of QVEMF and its benefits above was too good to be true. Surely merely accepting the Weak Quantifier Variance thesis and QVEMF can't suffice to simultaneously banish nominalists' Quinean indispensability worries (by saying mathematical objects literally exist) and traditional platonists' arbitrariness worries (by saying that mathematical posits introducing new objects for study can express truths without selecting structures

[14]Technically appeal to the metaphysical semantics lets Sider eliminate the 'in virtue of' notion above from his theory, and restate completeness as follows, "New completeness: Every sentence that contains expressions that do not carve at the joints has a metaphysical semantics."

[15]Note that this difference in terminology doesn't reflect a commitment by Sider to only use the most fundamental quantifier sense when doing philosophy or even metaphysics. He also accepts that one sometimes does philosophy using less fundamental quantifier senses. It is merely a pure terminological difference.

from within some largest hierarchy of sets/total mathematical universe that just happens to stop somewhere).

With the Sideran framework above in place, we can express this incredulity crisply by saying adopting QVEMF merely pushes the bump in the rug because although it solves the above problems, our original choice of evils between platonism's arbitrariness and nominalism's indispensability problems exactly re-arises when we ask whether mathematical objects are fundamental (rather than merely asking whether mathematical objects exist).

Specifically one might argue that any version of QVEMF which avoids the traditional platonism's arbitrariness problem suffers from a grounding problem just as bad as the nominalist's indispensability problem as follows.

Accepting QVEMF requires treating all logically coherent choices of pure mathematical posits on par. So the proponent of QVEMF must either say there are fundamentalia corresponding to *all* (human expressible) logically coherent pure mathematical posits or *none*.

If the proponent of QVEMF takes a line analogous to platonism and says the former we get back traditional platonism's arbitrariness problem. For the fundamentalia are, on Sider's account, identified with the objects that exist in a particular quantifier sense (the most fundamental). Therefore, we could add a mathematical structure which isn't among the fundamentalia by adding a new structure consisting of all classes of objects which exist in the fundamental quantifier sense[16]. Thus a universe of plentiful mathematical fundamentalia must stop somewhere. But, on the other hand, it seems like any particular stopping point for this plentiful universe of mathematical fundamentalia. Note that this same argument literally allows us to derive

[16]As before new structure would not already have been instantiated within the original mathematical universe because it must be strictly larger than that universe by Cantor's diagonal argument that the power set of a set is always larger than the original set.

a contradiction from the assumption that all possible mathematical structures are fundamental (the new structure created would be such a possible structure) but if (contra Sider) we can't actually refer to the fundamental quantifier sense we don't get an outright contradiction from the assumption that all mathematical structures we could posit are fundamental but we still must accept all the arbitrariness problems.

So, it seems that (on pain of jettisoning their only advantage over traditional platonism) the proponent of QVEMF is forced to say no mathematical objects are fundamental (a position analogous to nominalism). But then they are subject to a version of the indispensability challenge framed in terms of grounding: what can ground the truth of scientific statements which quantify over mathematical objects if there are no mathematical objects among the fundamentalia? Unfortunately, at first glance, the problem of providing a systematic grounding for scientific statements looks very structurally similar to, and no easier than, that of providing a nominalistic paraphrase for them.

Thus, one might fear, providing nominalistic grounding for the truths expressed by our best physical theories is no easier than providing a nominalistic paraphrase for these theories. In this case, however much reason the history of failures to answer the Quinean indispensability argument provides for thinking we can't adequately nominalistically paraphrase our best scientific theories, this history provides the same reason for thinking that (contra our QVMF theorist) the truths expressed by these theories cannot be nominalistically grounded[17].

[17]Admittedly, it's not too hard to answer grounding and paraphrase challenges as it applies to pure mathematical statements, and certain kinds of simple applied mathematical challenges. For they can simply re-purpose nominalists' existing logical regimentations of these statements as stories about grounding. For example in [16] Hellman nominalistically regiments each pure mathematical sentence ϕ about the natural numbers, with (simplifying slightly) a claim about logical necessity of the form $\Box$[If there are objects with the intended structure of the natural numbers i.e,. objects related by some relations as per the axioms

3.3. **Formulation of the Revenge Argument.** Putting this all together we can informally articulate a 'Revenge of Quinean Indispensability' argument against QVEMF (or at least any version of QVEMF which would avoid traditional platonism's arbitrariness problems) as follows.

(1) All facts are grounded in facts involving fundamental objects, relations, logical and perhaps modal vocabulary. (So, for example, demands for grounding are legitimate and there are no infinite descending chains of grounding)[18]
(2) If the Quantifier Variance explanation of mathematicians' freedom (QVEMF) is true, then there is nothing metaphysically special about the collection of pure mathematical structures we currently employ; For instance, it would be exceedingly arbitrary to insist that just the structures we currently employ have the property of being fundamental (or of having copies in fundamental structures).
(3) So if QVEMF then no pure mathematical objects are fundamental. Since, as discussed above, assuming that all possible mathematical structures we could posit are fundamental would would give us back traditional Platonism's arbitrariness worries.

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of second order Peano Arithmetic (which categorically describe the natural numbers), then ϕ holds of these objects.] The proponent of QVEMF can accept the surface logical form of S but instead take Hellman's paraphrase strategy to show how the truth of S (and the existence of any mathematical objects it quantifiers over) can be seem as systematically grounded in the nominalistic fact ϕ. And [5] suggests a way of conceptually simplifying these paraphrases.
However, it's much less clear that one can adequately ground statements of applied mathematics (especially ones that make complex claims involving magnitudes like length and charge or probabilities). One might think that whatever blocks classic nominalists from systematically (nominalistically) paraphrasing contemporary physical theories involving objective probability (and the like), will also block Quantifier Variantist from providing an adequate grounding for such claims.

[18]c.f. Sider's Completeness thesis: "Completeness Every nonfundamental truth holds in virtue of some fundamental truth.".

(4) Only fundamental objects can figure in fundamental facts. (This follows from Sider's Purity thesis "Fundamental truths involve only fundamental notions." [27].)
(5) So, if QVEMF, then no fundamental facts involve pure mathematical objects.
(6) So if QVEMF all facts (including those expressed by scientific statements which seem to quantify over mathematical objects) are ultimately grounded in nominalistic facts, i.e.m facts about relations between non-mathematical objects, modal facts about logical possibility and the like.
(7) If known scientific facts could be adequately grounded in nominalistic facts then they could be nominalistically paraphrased to solve the traditional Quinean indispensability problem. (This is claim is motivated the second half of the 'bump pushing' intuition above.)
(8) So nominalists' historic failure to solve the classic Quinean indispensability problem provides strong reason to think the analogous grounding problem cannot be solved – and thus that QVEMF is false.

Some possible ways of resisting the revenge of Quinean indispensability argument above are obvious. For example, the existing neo-carnapian advocates of QVEMF will likely reject (1) along with the very concept of metaphysical grounding. And philosophers who aren't worried by the Quinean indispensability problem for nomianlism will, presumably, reject (8) – denying that the current state of philosophical play provides strong reason to think either nominalistic paraphrase or nominalistic grounding of known scientific facts is impossible.

But to defend my claims about the promise of combining QVEMF with metaphysical realism above (that it can improve on both nominalism and traditional platonism), I must show that the 'Revenge' argument can be resisted in a different way. I must show that philosophers who take both grounding and the Quinean indispensability argument against traditional nominalism seriously can resist it. In the next sections I will discuss two approaches to doing this:

- arguing (contra 8) that the grounding challenge might be significantly easier to solve than the classic indispensability challenge (as traditionally understood),
- showing that (contra the inference from 2 to 3) QVEMF is compatible with some of the pure mathematical structures we currently employ being fundamental and thereby metaphysically special.

However I don't mean to suggest that these are the only plausible options[19].

4. Defending Nominalistic Groundabaility

Let us begin by considering the first option above: resisting the inference from a history of failure to nominalistically *paraphrase* certain scientific theories to pessimism about nominalistically *grounding* these theories. I will

[19]For example, one could also resist this argument by rejecting claim 4, the Purity thesis that fundamental facts must only involve fundamental notions. Saying the above would clearly involve some divergence from the Siderian framework above. For claim 4 was Sider's purity thesis. But perhaps we can separately motivate this rejection by considering the puzzles about what grounds grounding facts [15] which can be avoided by taking grounding facts to be fundamental.

If we reject Sider's purity thesis we can say that mathematical objects are not metaphysically fundamental (and ground the truth of internal statements about then in facts about pure logical possibility as Hellman suggests), but still ground scientific facts in facts involving mathematical objects however the platonist would. For example we can ground scientific facts about mass in facts involving, say, real numbers and a mass/mass ratio relation relating objects/pairs of objects to real numbers just as standard platonist would. However this option would still seem to require treating some pure mathematical objects as special (by saying fundamental physical magnitude facts are/are grounded in relations to these objects), if we are to avoid massive redundancy and the return of the issues for plenitudinous platonism noted above. Thus I don't see it providing many advantages over the Agnostic Platonist option discussed below.

discuss two (compatible) ways of motivating this idea, and then briefly invoke a radical view (with some attractions) on which the above inference certainly fails.

4.1. **Grounding Easier than Paraphrase.** First, one might argue that nominalistically grounding facts about applied mathematics is easier than nominalistically paraphrasing such facts in a few important ways.

For one thing, giving a nominalisitic paraphrase of a scientific theory ϕ mentioning mathematical objects has generally been taken to require providing a *single* (logically regimented) sentence with the same intuitive meaning which does not. However it seems independently attractive to say that we can ground a single fact in infinitely many other facts. For example the fact that $P\&Q$ can be grounded in both the fact that P and the fact that Q, and the fact 'there is a star' can be partly grounded in the the fact that each particular start exists, even if there are infinitely many stars. So as Stanford Encyclopedia[6] puts it:

> " It seems that there are cases in which a single fact is grounded in a plurality of facts (e.g., [$p\&q$] (the fact that $p\&q$) is grounded in [p],[q]), so we can think of the logical form of grounding statements on the predicate view as follows: [p] is grounded in Δ, where Δ is a plurality of facts."

Admittedly some philosophers[14] addressing the classic Quinean indispensability challenge do allow a nominalistic paraphrase to consist in a countably infinite collection of nominalistic sentences, provided that these can be algorithmically listed. But the considerations about size above considerations would seem to suggest that the facts about fundamentalia grounding the truth of a given fact could be uncountably infinite (just substitute 'spatial point', for star). And such an algorithmically listable infinite collection of sentences couldn't contain infinitely many atomic predicates.

Second, it is widely thought that a systematic logical regimentation of a person's natural language statements cannot employ infinitely many different atomic predicates and relations, because a language with infinitely many atomic predicates would be unlearnable[11]. But once we replace the nominalists' task of describing the true logical structure of our scientific beliefs with the task of saying what facts about fundamentalia ground the truth of these beliefs, (I think) this argument from learnability no longer applies. So it's not clear that grounding facts couldn't involve infinitely many atomic predicates.

For there is (prima facie) no reason to assume that human beings must be able to learn distinct names for all the atomic properties which would be used in a maximally metaphysically joint carving language.

Perhaps one could argue from the above constraint on learnability to this claim about metaphysical hypotheses, if one thought that a philosopher's explanation for what does/could ground a fact [ϕ] had to take the form of a claim like, 'The fact that ϕ grounded in the fact that ψ'. However, we don't (and shouldn't) currently take this to be a genuine constraint on metaphysicians expressing theories of grounding, as the idea that some facts have infinitary grounds above highlights. It's enough for the nominalist to somehow indicate a pluarlity of facts which does the grounding. They can say something like 'the fact that such-and-such spatial region has property P is grounded in the pluarality of facts that, for each point x in that region, x has property P'. Furthermore there seem to be good reasons that we *shouldn't* constrain the acceptable statement of metaphysical hypotheses about grounding and fundamentality as above.

Considering extant theories of grounding and fundamentality also discourages applying the above Davidsonain constraints to questions about grounding. It seems actively unreasonable to rule out (in general, and in advance) the possibility that a more metaphysically joint carving language would contain infinitely many atomic relations and would thus not be human learnable. For there some traditional, intuitively meaningful and important metaphysical hypotheses which could not be stated under this condition. For example consider Leibntizian hypotheses where the there's an infinite descending chain of different kinds of progressively more fundamental objects and properties obeying different laws. One may or may not find such hypotheses about grounding and fundamentality plausible. But they seem perfectly intelligible (as intelligible as metaphysical hypotheses usually are) and it seems unreasonable to rule to rule them out as contrary to some kind of rules for metaphysical discussion.

Thus, to summarize, there seems to be no reason to assume that humans could speak a language which has vocabulary for all and only metaphysically fundamental concepts. Accordingly we have no reason to forbid the nominalist account of scientific grounding from making use of infinitely many atomic relations, infinitely long sentences or non-computable infinite collections of sentences. To briefly motivate the usefulness of such infinitary grounding note that one major challenge faced by hard road nominalists concerns how to logically regiment statements about physical magnitudes alike mass and charge (in a way that accommodates intuitions that objects can stand in arbitrary precise mass or charge ratios even in possible worlds with very few objects in total)[23, 12]. And in *Mathematics Without Numbers* Hellman actually provides an example of how one might think about physical magnitude facts as grounded in (infinitary) facts about the application of an infinite number of nominalistically acceptable physical magnitude properties and relations (i.e., ones that don't take mathematical objects as relata) [20][21]

4.2. **More Resources.** Second, one can argue that (even if nominalistic paraphrase is not intrinsically easier than nominalistic grounding) the nominalistic grounding problem is easier to solve than the (notoiously intractable) indispensability problem for traditional nominalists who reject all abstract objects. Specifically, one might suspect that many nominalists trying to provide paraphrases for scientific theories were working under significant constraints which the proponent of QVEMF need not accept. And if this is so, it provides reason to resist the inference from nominalists' failure to find a paraphrase they consider acceptable to the impossibility of providing nominalistic grounding (in the sense relevant to QVEMF).

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[20]See section 3.4 of [16] and note Hellman's explicit comment that complete success in this project would not suggest that we could remove quantification over mathematical objects from our best scientific theories.

[21]Admittedly this style of response to the Revenge of Quinean indispensability challenge is somewhat (structurally) similar to a Melia's defense of nominalism against Quine's classic indispensability challenge in [22]. Melia motivates the idea that "we should not always believe in the entities our best physical theory quantifies over" because quantifying over mathematical objects is just a tool to let finite creatures like ourselves express claims which less limited creatures would express by asserting infinite conjunctions/disjunctions of nominalistic sentences (e.g., a claim about the number of planets around the average star might abbreviate an infinite disjunction of nominalistic descriptions of specific universe states).

And an influential line of criticism maintains that if we accept Melia's proposal — or in any other way drop the requirement that someone engaged in ontology state their best total theory of the world without quantifying over any objects they want to deny exist (in the sense relevant to the ontology room) – then we get a scenario where 'anything goes' as regard to ontology. That is, we loose any concrete grip we may hope to have had on how to settle ontological questions – and thereby perhaps any grip on what questions of traditional ontology mean. I'm not sure whether this criticism ultimately works against classic nominalists like Melia. For the the inference from, ' If $\neg P$ then we don't have a coherent and fruitful grip on the project of philosophical ontology' to 'P' can seem like a case of unjustified wishful thinking.

But even if this argument cut ice against a nominalist like Melia, we should note that the quantifier variantist who rejects mathematical fundamentalia and demands for finitary grounding has special tools for answering it which the nominalist does not. For, philosophers are already independently working on a theory of grounding and formal constraints on when one thing can be said to be grounded in another, and the Quantifier Variantist can say that this prevents it from being the case that 'anything goes' with respect to grounding.

For many philosophers are inclined to accept nominalism about mathematical objects because they have a general resistance to accepting 'strange', (i.e., necessary, abstract and/or non-material) objects. Thus when providing a nominalistic paraphrase in response to Quine's challenge, they will want to avoid quantifying over these objects as well.

In contrast, QVEMF theorist's motivation for saying that no pure mathematical objects are fundamental doesn't commit them to a general rejection of strange objects (in the sense above) as fundamentalia. They are motivated by a very specific features of mathematical practice (namely, mathematicians' taking themselves to be free to adopt arbitrary logically coherent conceptions of pure mathematical structures) to say that languages talking in terms of different pure mathematical structures are metaphysically on par, and no pure mathematical structures can be fundamental.

But no analogous freedom is claimed by physicists in their practice of talking in terms of strange objects such as events, electromagnetic fields, the wave function in Quantum Mechanics (if we construe this as something physically real), or the 'space' in which the wave function lives. Thus the proponent of QVEMF has no reason to deny that some of these objects could be fundamental. Hence they are free to use a much a wider range of non-mathematical objects in their grounding story than most traditional nominalists would be happy using in their paraphrase story[22].

[22]Relatedly, one might argue that philosophers giving a traditional 'hard-road' response to the Quinean Indispensiblity challenge (by providing a paraphrase for their best scientific theories) are likely to try to write paraphrases using notions Quine would accept. Thus, they are likely to use only first order logical quantification, and to avoid use of modal vocabulary (like a notion of logical possibility/coherence. In contrast, when QVEMF theorist attempts to explain how applied mathematical facts could be grounded in facts about relations between non-mathematical objects plus facts about logical possibility, they are free to use modal notions like logical coherence/possibility. For a menu of options for how such modal vocabulary can be useful see chapter 3 of [17].

To support the idea that allowing such such physically weird metaphysical fundamentalia can make the nominalists' task easier, note that avoiding quantification over events in probability statements is often cited as the largest sticking point for nominalist trying to answer indispensability worries[21].

5. Agnostic Platonism

The second, and more interesting, option I want to consider is what I'll call Agnostic Platonism. Suppose that we grant that the history of debate over Quine's indispensability argument suggests some mathematical objects are among the fundamentalia. Proponents of the QVEMF can still resist the Revenge of Quinean Indispensability argument above by rejecting the inference that all coherent conceptions of mathematical objects must be metaphysically (as opposed to merely mathematically) on par, and thence the argument that no mathematical objects can be grounding fundamental.

In slogan form, someone who accepts agnostic platonism would say: maybe some mathematical structures are metaphysically special, but mathematicians don't care which ones those are, and they don't need to care in order to reliably form true mathematical beliefs and satisfy the epistemic aims of the project of pure mathematics!

Allowing (in response to indispensability worries) that some mathematical structures may be metaphysically fundamental might seem to raise access worries (over and above the access worries about access to facts about logical coherence which the QVEMF theorist already faces[23]). For although these worries can suggest the fundamentalia plausibly include *some* mathematical objects, we don't know (and perhaps can never know) *which*[24].

[23]See [4] for an argument that these access worries about logical coherence are solvable.

[24]Perhaps grounding considerations motivate thinking that *some* collection of mathematical objects which are sufficiently plentiful and richly structured to do certain work in applied mathematics exist fundamentally. But, as has often been remarked (For example,

But the agnostic platonist avoids this access problem by saying that getting mathematics right doesn't require guessing which mathematical structures are among the fundamentalia. Note that this idea (that reliably speaking the truth in mathematical ordinary language doesn't require knowing the right answer to corresponding metaphysical questions about fundamental ontology) mirrors what it is natural to say about our knowledge of holes. It may turn out to be the case that some particular hole-like notion (maybe the topological notion of holes) will be used in physics, but construction workers can draw the line where they want with regard to hole boundaries and reliably speak the truth without having to take any such stance regarding fundamental metaphysics. One might object that a similar access worry arises with regard to metaphysicians' knowledge of which mathematical structures are grounding fundamental. However, we can answer this access worry by noting that there's no access to account for. Metaphysicians don't even *appear* to know very much about which mathematical structures are metaphysically fundamental.

Now a reader sympathetic to classic set theoretic foundationalism might object: how we can endorse the arbitrariness based criticism of set theoretic foundationalism in section 2 while advocating Agnostic Platonism about mathematical fundamentalia without hypocrisy? For one might worry that dividing up mathematical objects into those with fundamental existence vs. those without is just as arbitrary as saying that the hierarchy of sets just happens to stop at a certain point. And isn't being committed to arbitrariness in which mathematical objects are fundamental just as bad as being committed to arbitrariness in size of the total mathematical universe?

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see [8]) indispensability considerations don't seem to justify belief in any particular mathematical structure as different mathematical structures seem capable of doing the same work in regimenting/grounding our physical theories.

Even if this charge of hypocrisy were correct, I think the Quantifier Variantist view advocated above would still be an improvement on classic set theoretic foundationalism. For the arbitrary joint posited by the agnostic platonist doesn't constrain acceptable mathematical practice, whereas that posited by the set theoretic foundationalist/plenitudinous platonist does. The agnostic platonist need not admit any limits on which logically coherent pure mathematical structures mathematicians could choose to talk in terms of. For they don't think mathematicians can only introduce or study structures which are grounding fundamental. In contrast, the plentudinous platonist takes there to be a total mathematical universe (e.g., the hierarchy of sets), and holds that any conception of a pure mathematical structure mathematicians could legitimately adopt must have an intended model within it.

However, I will now sketch a more aggressive defense against this charge of hypocrisy. If the other assumptions needed for the Revenge of Quinean Indispensability Argument hold (i.e., we need to provide grounding, and mathematical objects appear indispensable to that task) then it seems that everyone, not just the agnostic platonist, must admit that certain mathematical structures are special in that they play a role in grounding non-mathematical facts about the world (e.g., maybe length reflects a fundamental facet of reality and length facts require grounding in the real numbers).

So agnostic platonism still has the advantage that it only requires us to posit that one special joint in the space of coherent conceptions of mathematical structures (specifying which particular mathematical structures play a role in grounding and/or constituting particular applied mathematical facts, e.g., facts about events and probability, or lengths) where the classic set theoretic foundationalist is committed to two positing two joints in reality (this joint, plus the joint determining where the hierarchy of sets happens to stop).

That is, both philosophers will be committed to facts like 'the pure mathematical objects which play roles in grounding physical facts are exactly the real numbers and three layers of sets over them'. But the set theoretic foundationalists will also be committed to a fact like 'the hierarchy of sets just happens to stop at X point' (where that point is usually taken to occur way above the point where all sets used in physical theories exists/what is needed to contain models for all mathematical structures used in physics).

Moreover, it seems more plausible that facts about the fundamental laws of physics might provide a, as yet undiscovered, principled division between those mathematical objects which play a role in grounding applied mathematical facts and those which don't, than it does that some choice of a height for the hierarchy of sets will turn out to be principled[25].

Thus, to summarize, I think the (admittedly prima facie strange) idea of saying that, although mathematicians can introduce any pure mathematical structure they like, some pure mathematical structures are metaphysically special and instantiated by objects which are grounding fundamental is more appealing than it first seems.

[25]Indeed, one might argue as follows. Applied mathematics hasn't seemed to motivate a unique choice of which mathematical structures exist, because (from a traditional platonist point of view) the total collection of mathematical objects must do two jobs. It must make sense of applied mathematics *and* everything we could study in pure mathematics. Given this goal, it has seemed natural to consider both, e.g., both a free standing real number structure and a copy of the real numbers within various larger structures, like the hierarchy of sets (containing objects for pure mathematical study), as candidates for mathematical reference within our best physical theories. And there's no uniquely natural choice of a collection of mathematical objects which does both jobs.
However the agnostic platonist does not expect fundamental mathematical objects to do both these jobs. (As noted above) they can take the truth of existence claims about pure mathematical objects to be grounded in something like facts about logical possibility. Thus it seems more plausible that whatever aspects of our best physical theories make appeal to some fundamental mathematical objects indispensable (if such there are) should suggest a unique most natural collection of mathematical structures to take to be grounding fundamental.

6. A Problem For Easy Road Nominalists

Before concluding, let me quickly point out a way in which the considerations above generalize.

Points 7 and 8 in the 'Revenge of Quinean indispensability' argument above amount to a general argument that we should expect mathematical objects to be grounding indispensible. And this creates problems for 'Easy Road' nominalists (i.e., nominalists who respond to the Quinean indispensability argument by rejecting the demand that it must be possible to literally state ones best theory)[26].

For easy road nominalists often accept the historically motivated pessimism about nominalistically regimenting our best scientific theories in (7). Indeed, I take such pessimism to be one of the major drivers of interest in easy road nominalism. And (although I have tried to raise doubts about 6) the point labled 6* below seems indisputable by anyone who accepts the general idea of grounding.

Revenge of indispensability argument against nominalsism

- 6* If mathematical nominalism is true all facts (including ones we would normally state by apparently quantifying over mathematical objects) are ultimately grounded in nominalistically acceptable stuff like relations between non-mathematical objects, modal facts about logical possibility etc...
- 7 Any adequate story grounding applied mathematical facts in nominalistically acceptable stuff could (easily) be transformed into a nominalist paraphrase for these facts.
- 8 So (by 7) nominalists' historic failure to provide an adequate nominalistic paraphrase for certain scientific statements (as per the classic

[26]Thanks to an anonymous reviewer for pointing this out.

indispensability challenge) provides strong reason to think no adequate nominalistic grounding for these statements is possible.

9* So nominalists' historic failure to solve the classic Quinean indispensability problem provides strong reason to think the analogous grounding problem cannot be solved – and thus that QVEMF is false.

Thus Easy Road nominalists seem to face a serious worry as follows, and will likely want to reject (8) (whether on the grounds I've advocated here or for other reasons).

7. Conclusion

In this paper I reviewed some appeals of using Quantifier Variance to explain mathematicians' freedom to introduce new pure mathematical structures for study. I noted that (although its advocates have traditionally been metaontological anti-realists) this explanation for mathematicians' freedom is *prima facie* compatible with metaontological realism.

I then developed a 'Revenge of Quinean Indispensability' problem for the quantifier variance explanation of mathematicians' freedom, which arises when we ask whether any mathematical objects are fundamental. The existence of this problem might seem to show that the Quantifier Variance explanation of mathematicians' freedom is ultimately off limits to metaontological realists (who are more likely to take questions about grounding seriously). However, I argued that a number of promising routes are available for solving this problem, including some which are compatible with metaontological realism.

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