The Mathematical Nominalist's Real Problem With Physical Magnitudes

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Abstract

A key reason for thinking mathematical nominalists can't answer the Quinean indispensability argument concerns difficulties nominalistically paraphrasing physical magnitude statements. In this note, I'll argue that nominalists who accept certain notions from the literature on potentialist set theory can avoid these difficulties by deploying two cheap tricks. Doing this lets us answer Quine's original challenge, although philosophical concerns involving reference, metaphysical possibility and grounding remain.

1 Introduction

Quinean indispensability arguments challenge mathematical nominalists to state their best scientific theory without quantifying over mathematical objects. A common motivation for pessimism about answering this challenge [7] [3, 4, 2] concerns difficulties formalizing physical magnitude statements (i.e., statements about lengths, temperatures and the like).

In this paper, I'll argue that a nominalist who accepts certain modal notions (independently motivated by the literature on potentialist set theory), can plausibly overcome these difficulties sufficiently to answer Quine's original challenge by deploying two cheap tricks. However certain philosophical concerns about reference, metaphysical possibility and grounding remain.

In §2 I'll quickly review relevant philosophical literature leading up to the key 'sparse magnitudes' problem I aim to solve. This problem threatens to prevent even nominalists who accept the existence of spatial paths/points (like

Hellman[4] and Field[3]) from adequately formalizing physical magnitude statements. In §3 I'll describe my pair of cheap tricks. And in §4 I'll note some limits to what this proposal accomplishes.

2 Setting up the Sparse Magnitudes Problem

Quine's indispensability argument famously challenges the mathematical nominalist to state a formalized version of their best total theory without quantifying over objects (like the numbers) which they don't believe in. To make this demand for formalization concrete, I'll consider whether one can 'adequately translate' a Platonist scientific theory T by producing a nominalistic paraphrase P(T) which is true at exactly the same metaphysically possible worlds where *the Platonist thinks* T is true.

A range of views in the literature on potentialist set theory let us talk about how it would be logically or 'interpretationally' possible to supplement the objects that actually exist with others satisfying certain axioms (c.f., Hellman's logical possibility together with plural quantification[4, 5], Berry's structure preserving/conditional logical possibility[1], Linnebo [6] and Studd's [9] interpretational possibility together with plural quantification).

So a natural question is whether we can use the same logical machinery to create (modal) if-thenist paraphrases of applied mathematical statements. Can we capture the non-mathematical content of a Platonist scientific theory ϕ by a sentence of approximately the following form?

> Necessarily, if there are (in addition to all the physical/nominalistically acceptable objects) extra objects related in the way the Platonist takes mathematical objects to be related by mathematical relations, then ϕ is true of objects.[1]

[1]Note that the mere material conditional involved in the claim above 'if there are objects

For example, we might translate the Platonist claim 'There's a (non-empty) set of critics who admire only each other' as saying something like the following.

> Necessarily, if there are new objects playing the role of sets of critics, then one of these objects contains (under some membership relation $\mathscr{E}$) a non-empty collection of critics who admire only each other.

Crucially, all the versions of set theoretic Potentialism mentioned above accept logical tools powerful enough to say that there are sets corresponding 'all possible ways of choosing' from physical objects that form the ur-elements for this hierarchy of sets.[2] Thus they can assert that there are objects with the intended structure of a full width hierarchy of sets with ur-elements (up to, say, $V_{\omega+\omega}$). And they can provide if-thenist paraphrases of applied mathematical claims using set theory with ur-elements.

However, a problem arises when we consider physical magnitude statements. When formalizing a theory like Newton's law of gravity, the Platonist can appeal to a length function which pairs each spatial path with its length-in-meters (a certain real number). And the nominalist must simulate or replace such talk of a length function where it appears.

As noted by philosophers like Field in [3], appeal to measurement theoretic uniqueness theorems suggests an answer to this problem (as regards length).

For when certain assumptions (which I'll call the claim that space is richly instantiated[3]) hold, we can give a definite description which picks out the Pla-

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satisfying such and such Platonist antecedent then...' will generally be trivially true from a nominalist perspective, because its antecedent is false. Appealing to the modal notions from potentialist set theory above (of interpretational possibility/logical possibility given the existence of certain objects etc.) lets us avoid this problem, by talking about what would have to be true if the objects the nominalist accepts were (in some relevant sense) supplemented with additional objects.

[2]So, for example we might say that some objects have the intended structure sets of critics (under some otherwise unused relations S and $\mathscr{E}$ apply like 'set' and $\in$) to by saying 'There are new objects satisfying a predicate S(u) and for any collection xx of physical objects there is a u satisfying $S(u)$ such that $\forall y \in xx$ we have $\mathscr{E}(u, y)$'.

[3]Specifically, we can prove the uniqueness claim above holds whenever the following three

tonist's length-in-meters function (among all other functions from physical objects to real numbers) by specifying that it assigns length 1 to some canonical path and respects the following pair of nominalistic relations:

- $\leq_L$ 'path p_1 is at least as long as path p_2'
- $\oplus_L$ 'the combined lengths of path p_1 and p_2 together are equal to the length of path p_3'[4].

Thus, we have a formula ψ which picks out the Platonist's length-in-meters function at all words where length is richly instantiated. So at all such possible worlds a Platonist sentence $\phi(l)$ (in the language of set theory with ur-elements with l being a name for this length function) will be true if and only iff the corresponding nominalist sentence $P^*(\phi)$ (below) is true.

> $P^*(\phi)$ 'Necessarily if there are objects satisfying our description of the hierarchy of sets with ur-elements $V_{\omega+\omega}$ then $(\exists f)(\psi(f) \wedge \phi[l/f])$'

Thus one might hope that Platonist appeals to length relations can be harmlessly replaced by the strategy above and that (as Field suggests in [3]) Platonist talk of mass, charge etc. functions could be handled similarly.

However, a difficulty which I'll call the Sparse Magnitude problem (and attempt to solve in this paper) arises. For, although lengths are plausibly richly instantiated in our world, it's not clear that they're richly instantiated at all

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principles (which all happen to be statable in the language of set theory with ur-elements) are satisfied.

Closure Under Multiples: Given a path x, there are paths y with lengths equal to any finite multiple of the length of x.

Archimedian Assumption: No path is infinite in length with respect to another, i.e., if $x \leq_L y$ then some finite multiple of x is longer than y (i.e. there's a path shorter than y, which can be cut up into n segments each of which has the same length as x.

Relational Properties: The relations $\leq_L, \oplus_L$ have the basic properties you would expect from their role as length comparisons.

My presentation follows [8].

[4] I will say a function $l(x)$ respects $\leq_L, \oplus_L$ just if for all paths a, b and c $a \leq_L b \iff l(a) \leq l(b)$ and $\oplus_L(a,b,c) \iff l(a) + l(b) = l(c)$.

metaphysically possible worlds. And other physical magnitudes, like mass and charge, don't even seem to be richly instantiated in the actual world. Indeed, as Eddon puts it [2]:

> It seems possible for there to be a world, w_1, in which a and b are the only massive objects, and a is twice as massive as b. It also seems possible for there to be a world, w_2, in which a and b are the only massive objects, and a is three times as massive as b. Worlds w_1 and w_2 are exactly alike with respect to their patterns of [how the relations 'less massive than' $o_1 \leq_M o_2$ and $\oplus_M(o_1, o_2, o_3)$ 'combined mass of a + mass of b = mass of c' apply]. And thus they are exactly alike with respect to the constraints these relations place on numerical assignments of mass. ... So it seems we cannot discriminate between the two possibilities we started out with.

These considerations threaten to block the above nominalist paraphrase strategy by showing that length is a special case. They suggest that other physical magnitudes (like mass) can't be pinned down in the same way that length can, and perhaps that the values of physical magnitudes doesn't supervene on facts about how *any* finite list nominalistic relations) apply[5].

3 A Solution – In a Sense

3.1 Four Place Relation

I'll now argue that we can solve the above sparse magnitudes problem by using two cheap tricks. Specifically, suppose the Platonist worries that object masses

[5]Thus version of Putnam's famous counting argument in [7] threatens to rearise, even for those nominalists like Field in [3] who avoid the specific concern about lengths he mentions by accepting the existence of spatial points or paths.

or any other physical magnitude (given by real numbers) can't be captured by any relations between nominalisticly acceptable objects.

First, I claim that if we (temporarily) assume that length is richly instantiated at all possible worlds, we can solve the sparse magnitude problem by using the relationship between length and mass to pin down a mass assignment property (and likewise for other physical magnitudes).

For example, the nominalist can pick out a correct mass function by appeal to a four-place relation between pairs of objects with masses and pairs of paths:

- $M(o_1, o_2, p_1, p_2)$ which holds iff the ratio of the mass of o_1 to the mass of o_2 is $\geq$ the ratio of the length of path p_1 to the length of the path p_2.

Although such a relation may not be very physically (or metaphysically) natural, it reflects a genuine nominalisticly acceptable fact about the world and suffices for our purposes. By the measurement theory results mentioned above, we can uniquely pin down the length function (up to a choice of unit), at all worlds where length is richly instantiated. Furthermore the claim that length is richly instantiated implies that, for any distinct real numbers r and r', there is a pair of paths p_1 to p_2 whose lengths stand in a ratio that's in the interval between r and r'.

Thus we can pick out the intended mass in grams function $\mathscr{M}$ (within our simulated hierarchy of sets with ur-elements) by saying that it assigns mass 1 to a suitable unit object and assigns mass ratios which bear the right relationship to the length ratios assigned by a correct length function. Specifically, we demand that any mass function $\mathscr{M}$ satisfy the constraint that if $\mathscr{L}$ is a length function respecting $\leq_L, \oplus_L$ then $M(o_1, o_2, p_1, p_2)$ holds iff $\mathscr{M}(o_1)/\mathscr{M}(o_2) \geq \mathscr{L}(p_1)/\mathscr{L}(p_2)$[6]. The latter condition ensures that $\mathscr{M}$ assigns mass ratios cor-

[6]Consider any $\mathscr{M}'$ that attempts assign the wrong mass ratio r' to a pair of objects o_1, o_2 with mass ratio r. Any such function will fail to honor the true $\mathscr{M}(o_1, o_2, p_1, p_2)$ fact relating the ratio between the masses of o_1, o_2 to the ratio of length between a pair of paths p_1, p_2

rectly, provided length is richly instantiated.

This, in turn, is enough to allow us to apply the paraphrase strategy discussed above to claims involving a mass function (and the same goes for other physical quantities).

Importantly, even if length isn't *necessarily* richly instantiated, the modal if-thenist paraphrase strategy described above still gives the correct truth-values in those worlds where length is richly instantiated.

3.2 Holism trick

Now what about the above assumption that length is *metaphysically necessarily* richly instantiated? This assumption seems unmotivated but, happily, we can eliminate it if (as currently appears to be the case) our best scientific theory implies that length is *actually* richly instantiated[7].

To see how, consider some such platonistically formulated theory T which implies that space is richly instantiated.

By the considerations above we can produce a partially accurate paraphrase $P^*(T)$ which gets the correct truth-value at worlds where length is richly instantiated, but may get the wrong truth value at other possible worlds.

We can also write a completely correct nominalistic paraphrase of the claim that space is richly instantiated (call this R)[8].

Then we can create a paraphrase which gets correct truth conditions for our theory at all possible worlds by simply writing the following conjunction.

P(T): $P^*(T) \wedge R$

such that $\mathscr{L}(p_1)/\mathscr{L}(p_2)$ falls between r and r'. And the existence of such a pair of paths is guaranteed by the assumption that length is richly instantiated, as noted above.

[7] A similar technique can plausibly be used to paraphrase physical theories that say that space is quantized because they tend to say that other physical magnitudes are also quantized. Thanks to REDACTED for this point.

[8] Note that this claim is statable using only set theory with ur-elements and the relations $\leq_L, \oplus_L$, so our basic modal if-thenist strategy suffices to paraphrase it.

At worlds where length is richly instantiated, $P^*(T)$ has the correct truth value by our initial point, and R is true at those worlds, so the above conjunction will have the correct truth value. And at worlds where space isn't richly instantiated R is false, hence so is our paraphrase. Thus, in both cases, our paraphrase has the intended truth value.

Thus, the nominalist plausibly *can* address the sparse magnitude problems sufficiently well to answer the classic Quinean indispensability argument.

4 Conclusion

In this paper I have argued that a mathematical nominalist (who accepts certain modal notions independently motivated by the literature on Potentialist set theory) can answer classic Quinean indispensability worries by deploying a pair of cheap tricks.

However, readers will likely find this response to Quine quite unsatisfying as a general defense of mathematical nominalism. First, the four-place relation M invoked above feels worryingly extrinsic, arbitrary and not physically natural — like something that shouldn't be metaphysically fundamental. Thus, nominalist can seem to face a grounding problem. What *does* ultimately ground the truth of physical magnitude claims (if not facts about M or a relation between physical objects and mathematical ones)?

Second, formalizing our best scientific theory as above leaves our (seeming) ability to make other scientific claims a mystery. For example, one might think we can meaningfully state mass claims conjoined with partial physical theories which *don't* imply that length is richly instantiated. But (prima facie) the nominalist paraphrase strategy I've discussed here doesn't let one formalize such statements.

Thus, arguably, the nominalist's real problem with physical magnitudes doesn't concern stating our best scientific theory but rather accounting for certain a priori philosophical intuitions about metaphysical possibility, reference and grounding. Philosophical explanation is the sticking point, not scientific explanation. Accepting this point may have various interesting philosophical consequences (e.g., reducing the appeal of indispensability arguments to hardcore naturalists) but I won't attempt to explore them here.

References

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