The Mathematical Nominalist's Real Problem With Physical Magnitudes

Sharon Berry

Abstract

Quine's indispensability argument challenges philosophers who don't believe in mathematical objects to state their best total scientific theory without quantifying over mathematical objects. A key reason for thinking the nominalist can't answer this challenge (going back to [11] but refined in the subsequent literature [7, 9, 6]) concerns difficulties nominalisticly paraphrasing physical magnitude statements (i.e., statements about lengths, temperatures and the like).

In this paper I'll note that some simple tricks plausibly let us avoid these difficulties with physical magnitudes, so far as answering Quine's original challenge goes. However, I'll suggest that a genuine problem remains. Overall, I'll argue that the real challenge which physical magnitude statements pose for nominalists concerns accommodating certain *philosophical* intuitions (about reference, metaphysical possibility and grounding) not literally stating our best total theory scientific theory.

1 Introduction

Quine's indispensability argument challenges philosophers who don't believe in mathematical objects to state their best total scientific theory without quantifying over mathematical objects. A key reason for thinking the nominalist can't answer this challenge (going back to [11] but refined in the subsequent literature [7, 9, 6]) concerns difficulties nominalisticly paraphrasing physical magnitude statements (i.e., statements about lengths, temperatures and the like).

In this paper I'll note that some simple tricks plausibly let us avoid these difficulties with physical magnitudes, so far as answering Quine's original challenge goes. However, I'll suggest that a genuine problem remains. Overall, I'll argue that the real challenge which physical magnitude statements pose for nominalists concerns accommodating certain *philosophical* intuitions (about reference, metaphysical possibility and grounding) not literally stating our best total theory scientific theory.

In §2 I'll quickly review the literature on the problems with nominalistically paraphrasing physical magnitude statements. In §3 I'll point out a formal trick that lets us solve these problems for the purpose of answering classic Quinean indispensability worries. In §4 I'll point out related reference and grounding worries that remain, and in §5 I'll conclude and discuss some possible consequences of accepting the picture sketched here.

2 History of the Problem

2.1 Putnam

In [11], Putnam provides an influential counting argument that (a certain kind of) nominalist cannot write logically regimented sentences which are "adequate for the purposes of science" because they cannot (appropriately) logically regiment certain statements about lengths.

In particular, Putnam targets a materialist nominalist, who believes in broadly material objects like sticks, stones and electrons, but not any immaterial objects like numbers or spatial points. He notes that many scientific theories are ordinarily stated by appealing to a physical magnitude function, like a length or mass function which relates physical objects to numbers. For the nominalist to capture the way we apply laws like Newton's law of gravity:

$$F = \frac{gM_aM_b}{d^2}$$

they must interpret length statements that have (something like) the following form

L_i: 'c is $q_0 \pm q_1$ times the length of d'[1]

where q_0 and q_1 are rational numbers[2]

Now Putnam argues that his nominalist cannot paraphrase their best scientific theory 'adequately for the purposes of science', for the following reason.

Intuitively, there are infinitely many different ratios which the lengths of a pair of sticks could (in principle) stand in to one, another while existing in a world with $n \geq 2$ or fewer materialistic objects. So, for each pair of length statements $L_i \neq L_j$ like 'The ratio between c and d is 3.2 ± 1 and 'The ratio between c and d in 4.1 ± 3.37', it should be metaphysically possible that the ratio between c and d is in one interval but not the other. So we shouldn't rule out the possibility that there are at most n material objects, but $\neg(L_i \leftrightarrow L_j)$ (for any distinct L_i and L_j as above). No statement of the following form (for distinct sentences L_i and L_j as above) is necessary[3] should be a necessary truth:

Finite Objects Conditional for L_i, L_j: 'If the number of individuals is at most n then $L_i \leftrightarrow L_j$'

However, we can show (by a counting argument) that for some i, j the associated Finite Objects Conditional must be a necessary truth, (assuming, as is commonplace, that all statements are formalized using only finitely many[4] relations $N_1, \ldots N_m$). Informally speaking, this holds because there are only finitely many distinct scenarios which the application of $N_1, \ldots, N_m$ to at most n objects can distinguish. But there are infinitely many distinct length ratio sentences, i.e., sentences of the form L_i for some i. Hence, by the pigeonhole principle some pair of distinct length sentences L_i and L_j must take on the the

[1] I mean this to abbreviate a corresponding claim where c and d are replaced by nominalisticaly acceptable definite descriptions or names of spatial paths (or other physical objects) with lengths.

[2] Note that by using only rational numbers, but including a margin of error, Putnam is able to approximate the claim that x has length r times the unit meter stick in Paris (though, of course, now the meter is defined differently) for any real number r to any degree of accuracy (while speaking a finite human-learnable language).

[3] More specifically, as Putnam points out, some such sentence will be a theorem.

[4] C.f [5]

same truth-value in all such scenarios. Thus, some Finite Objects Conditional with distinct L_i and L_j turns out to be a necessary truth despite the fact that Putnam's nominalist is committed to it's possible falsehood . Hence, Putnam's nominalist can't logically regiment all sentences of the form L_i in a way that accommodates the metaphysical possibility intuitions above.

In contrast, Putnam argues that a platonist who believes in spatial points (or paths) can adequately regiment physical magnitude statements in essentially[5] the following way. One can define the length ratio between path p_1 and p_2 by saying that the ratio of the length of p_1 to p_2 is r iff for every function f which respects[6] the following nominalistic relations

- $p_1 \leq_L p_2$ iff path p_2 is as long or longer than path p_1
- $\oplus_L(p_1, p_2, p_3)$ iff the combined lengths of path p_1 and p_2 together are equal to the length of path p_3

and satisfies certain other obviously necessary conditions for being a correct length function, we have $\frac{f(p_1)}{f(p_2)} = r$. Measurement theoretic uniqueness theorems ensure that if space satisfies the condition that length is richly instantiated then there is a unique function f satisfying the above conditions (and thus length-ratios are all well-defined). The assumption that length is richly instantiated is, roughly, a way of asserting that any path can be subdivided into n equal length sub paths (Note, we will later use the fact that such a condition can be nominalisticly stated[7].

One might wonder what becomes of the counting argument above which, we should note, was proved without any assumptions about nomainalism. Why

[5]Putnam actually defines a 'distance in meters' function which applies to pairs of points. My deviation from Putnam's approach is inspired by [12].

[6]The function $l(x)$ respects $\leq_L, \oplus_L$ just if $a \leq_L b \iff l(a) \leq l(b)$ and $\oplus_L(a,b,c) \iff l(a) + l(b) = l(c)$.

[7]See the appendix for comparison of Harty Field's way of nominalizing this statement in [7] vs. Hellman[9] and Berry's[2] modal if-thenist strategy.)

doesn't it cause problems for Putnam's platonist? Philosophers who accept spatial points have no (immediate) problem with accepting this theorem. For they will say that all the intuitively metaphysically possible scenarios considered above (where there are only *n material* objects but two target objects some length ratio ensuring $\neg(L_i \leftrightarrow L_j)$) involve infinitely many spatial points. Thus the antecedent of the Finite Objects conditional isn't satisfied by these scenarios. Thus there are infinitely many such scenarios distinguishable in terms of contingent facts about how some finite collection of relations apply to (the infinite number of total) objects in these intuitively metaphysically possible worlds with only n or fewer material objects.

2.2 Field, Hellman et al. on Physical Magnitudes

Hartry Field's influential nominalization of Newtonian mechanics in [7], responds to the problem Putnam raises by, in effect, pointing out that a nominalist who (unlike Putnam's target) does believe in spatial points can analyse physical magnitude statements using the same finite collection of relationships between spatial points Putnam does.

They can appeal to (approximately) the same finitely many nominalistic relations between spatial points that Putnam invokes to paraphrase length statements. And then they can appeal to the same measurement theoretic uniqueness theorem to show that at worlds where Putnam's assumptions about space hold (i.e., worlds where a nominalisticly stateable version of the Rich Instantiation condition above holds), length facts do supervene on facts about how these nominalistic relations (between spatial points) apply. When considering other physical magnitudes like temperature, Field suggests that we could use a similar strategy if we assume that temperatures are also richly instantiated.

Now in [7] Field uses a number of other clever tricks to argue that we can paraphrase scientific claims in a nominalisticly acceptable fashion. However, I won't review these details because they won't matter to the objection I'm going to consider, and because other styles of nominalist paraphrase would connect these dots differently[8]. For instance, modal if-thenism of the kind advocated by either Berry[2] or Hellman[9] tells us that we can give nominalisticly acceptable paraphrases by considering the possibility of augmenting the objects at the actual world with objects mimicing the platonic objects. However, both of these strategies only function if we can uniquely describe the behavior of the platonic objects in nominalistic terms. For instance, these strategies would let us nominalize a theory phrased in terms of the natural numbers, since we can categorically describe the natural numbers in second order logic[9] But, absent a proof of the kind Putnam alludes invokes in case of the length function (and Field adapts in [7]), it is not clear that this strategy can be applied to platonistic theories mentioning a mass function. For the kind of counting argument which Putnam makes (which we will see returns below) exactly calls into question whether facts about the platonists length/mass etc. functions suitably supervene on facts about the application of any finite collection of relations between objects the nominalist believes in [10].

[8]It is also interesting to note that Field's own criticism of Hellman's easier style of easy road nominalism concerns a failure to establish supervenience on facts about how finitely many nominalistic relations apply – supporting my sense that this is a key issue in the debate about the viability of hard road nominalism.

[9]It is similarly easy to use this strategy for nominalizing claims about sets of goats, since we can write down a sentence that uniquely pins down the structure of the sets of goats at every metaphysically possible world.

[10]As further evidence of the importance of this concern about whether one can pin down physical magnitudes sufficiently to paraphrase our best physical theories (using finitely many nominalistically acceptable relations), note that Field's main criticism of Hellman-style modal if-thenism in [8] is that it has to appeal to infinitely many atomic predicates to paraphrase physical magnitude statements, and that Hellman explicitly notes that suggestions about applied mathematics in [9] don't let us answer the Quinean indispensability worries but only a related grounding concern because he can't avoid appeal to infinitely many atomic relations.

2.3 Sparse Magnitude Objection

Considering physical magnitudes other than length raises a potential problem, which has been stated explicitly as an objection to Field's proposal[6]. Specifically, it seems some physical magnitudes can take on definite values which don't supervene on the kind of relations Field makes use of. As Eddon puts it:

> It seems possible for there to be a world, w_1, in which a and b are the only massive objects, and a is twice as massive as b. It also seems possible for there to be a world, w_2, in which a and b are the only massive objects, and a is three times as massive as b. Worlds w_1 and w_2 are exactly alike with respect to their patterns of [less massive] and [same mass] relations. And thus they are exactly alike with respect to the constraints these relations place on numerical assignments of mass. But if they are exactly alike with respect to the constraints these relations place on numerical assignments of mass, then it cannot be the case that these worlds differ with respect to the masses of a and b. So it seems we cannot discriminate between the two possibilities we started out with[11].

We can make Eddon's point more generally (as an argument that no analysis of a certain kind could work) by formulating it as a slight modification of Putnam's counting argument as follows. Consider mass ratio sentences of the form

M_i: The masses of objects 'c and d stand in ratio $q_{i_0} \pm q_{i_1}$'

By the intuition Eddon expresses above, it seems that, for each number $n \geq 2$ and mass ratio $r \in \mathbb{R}$, it is (metaphysically and, for large values of n, epistemically) possible that only n objects that have masses exist, and yet the

[11] This is stated as an objection to a nominalization attempt involving only two physical magnitude relations, but the same counterexample works if we add the third $\oplus$ relation considered above

objects c and d stand in mass ratio r. So for any distinct mass ratio statements M_i and M_j, it is possible for there to be at most n objects but $\neg(M_i \leftrightarrow M_j)$. So, by Putnam's argument, any attempt to regiment M_i using finitely many relations **between the objects that have masses** will fail.

In principle, one could address this problem by accepting a rich space of abstract platonic objects corresponding to all possible values of physical magnitudes which concrete objects can take on. For example [1] discuss invoking a rich mass space, containing abstract objects corresponding to all possible mass properties, where particles 'occupy' points in this mass space exactly when they have the relevant mass property. One can then appeal to finitely many mass ratio relations. For this space of (abstract objects witnessing) all possible masses will be richly instantiated enough to uniquely pin down assignments of a number to each mass objects. I personally think this is an appealing option for nominalists about mathematical objects who are motivated by specific considerations in higher set theory, and hence are not inclined to deny the existence of other abstract objects. However, most nominalists will not be satisfied with it.

3 The Real Problem with Physical Magnitudes

3.1 On Adequate Paraphrase

I will now argue that we can solve the above problem about nominalistically paraphrasing physical magnitude statements well enough to answer the classic Quinean Indespensability argument. As mentioned above, the modal if-thenist strategies of Berry and Hellman[2, 9] can attractively nominalistically paraphrase[12] a target platonist theory, provided we can write a sentence that completely pins down the behavior of the platonic objects used in our best theory

[12] Specifically, we can state a nominalistic version of a platonist scientific theory which is true at exactly the same metaphysically possible worlds where the platonist thinks the original is true.

in terms of their relation to nominalsitic objects. For instance, if the platonistic theory is formulated in terms of sets of spacetime points then we'd have to write down a sentence[13] specifying which such sets exist [14].

The concerns Eddon raises potentially cast doubt on our ability to uniquely pin down the behavior of the mass ratio function in terms of nominalistic facts. I now show how to address this problem (modulo a tweak to be explained below) sufficiently for the purposes of solving Quine's challenge to literally state our best scientific theory without quantifying over mathematical objects.

3.2 Four Place Relation Trick

To start with, let me note that if we (temporarily) assume that length is richly instantiated, then we can solve Eddon's problem by use distance ratios to nominalistically pin down other physical magnitudes.

Specifically, suppose the Platonist worries that object masses or any other property (given by real numbers) can't be captured by any relations between nominalisticly acceptable objects. The nominalist can respond by invoking a four place relation M between pairs of objects with masses and pairs of paths [15]]:

- $M(p_1, p_2, m_1, m_2)$ which holds iff 'the mass m_1 is as many times (or more) the mass of m_2 as the length of the path p_1 is to the length of the path p_2'.

Even though such a relation may not be very physically (or metaphysically)

[13]Technically, the sentence must restrict all its quantification to objects satisfying the finitely many nominalistic relations and the platonic relations specifying the structure being described. However, this will cause no difficulty.

[14]Specifically, we would provide a sentence $D(S, \in, P)$ such that if $D(S, \in, P) \wedge D(S', \in', P)$ then S', S and $\in, \in'$ have the same extensions (where P is the predicate for space time points, S for sets of such points and $\in$ membership).

[15]If you prefer to take length to relate pairs of spatial points rather than paths, as Field's strategy for nominalistcally stating rich instantiation conditions requires, we can replace each path with a pair of spatial points.

natural, it reflects a genuine nominalisticly acceptable fact about the world, and suffices for our purposes. By the measurement theory results mentioned above, we can uniquely pin down the length function (up to a choice of unit) using nominalist relations $p_1 \leq_L p_2$ and $\oplus_L(p_1, p_2, p_3)$, at all worlds where length is richly instantiated. Supplementing such relations with M suffices to pin down mass ratios by comparing them to length ratios[16]. This, in turn, is enough to allow us to apply Berry or Hellman's paraphrase strategy as discussed above.

Importantly, even if length isn't *necessarily* richly instantiated, the paraphrases produced by Berry and Hellman's modal if-thenist approach still give the correct truth-values in those worlds where length is richly instantiated.

3.3 Holism trick

Now what about the above assumption that length is *metaphysically necessarily* richly instantiated? This assumption seems unmotivated but, happily, we can eliminate it if (as currently appears to be the case) our best scientific theory implies that length is *actually* archimidean or satisfies various other such constraints which imply that it's richly instantiated (as a matter of physical law)[17].

To see how, consider some such platonistic theory T which implies that space is archimidean.

By the considerations above, we can produce a partial nominalisation (call it $P^*(T)$) which gets the correct truth-value at worlds where length is richly instantiated, but may get the wrong truth value at other possible worlds. Call

[16]Specifically, we demand that any mass function $\mathscr{M}$ satisfy the constraint that if $\mathscr{L}$ is a length function respecting $\leq_L, \oplus_L$ then $M(m_1, m_2, p_1, p_2)$ holds iff $\mathscr{M}(m_1)/\mathscr{M}(m_2) \geq \mathscr{L}(p_1)/\mathscr{L}(p_2)$. Note that any attempt to assign the wrong mass ratio r' to a pair of objects m_1, m_2 with mass ratio r can be ruled out by considering paths p_1, p_2 whose length ratio falls between that of r and r' and noting that $\mathscr{M}$ fails the above condition for a pair of paths such that $\mathscr{L}(p_1)/\mathscr{L}(p_2)$ falls between r and r'. The existence of such a pair of paths is guaranteed by the assumption that length is richly instantiated.

[17]For example, we can make the same argument below if our best theory implies that space is quantised, i.e., all distances between spatial points are multiples of some minimum distance.

this .

And we can write a completely correct paraphrase of the claim that space is archimedean nominalistically. For example Field does this this on page 39 of [7] [18].

Then we can create a paraphrase which gets correct truth conditions for our theory at all possible worlds by simply writing the following conjunction.

Paraphrase: $P^*(T) \wedge P(A)$

At worlds where length is archimedean, $P^*(T)$ has the correct truthvalue, by our initial point, and P(A) is always true, so our total paraphrase has the correct truth value. And at worlds where space isn't archimidean, P(A) is false, and so is the paraphrase. Thus, in both cases, our paraphrase has the intended truth value.

This strategy can be easily extended to handle properties taking values in $\mathbb{R}^n$ or $\mathbb{C}^n$] .

Thus, the nominalist plausibly *can* address the problems about physical magnitude statements raised by Putnam and Eddon sufficiently well to answer the classic Quinean indispensability argument.

4 Remaining Indispensability Worries

However, readers will likely find the above solution quite unsatisfying! I'll suggest that this is because further grounding and reference indispensability worries remain.

To clarify the nature of these worries, I'll briefly discuss some possible strategies for responding to them. But note that I'm ultimately agnostic about whether they can be solved. I don't attempt to solve them here or to argue

[18] It may be worth noting that he does this with an infinite schema. See appendix X for a note about how the Berry-Hellman strategy paraphrases this using a single sentence.

that they can't be solved (so the nominalist faces a serious indispensability problem relating to physical magnitudes, despite their ability to satisfy classic indispensability worries by applying my cheap trick) .

4.1 Grounding Indispensability

First, intuitively speaking, it feels worrying that kind of 4 place relation above (between paths with lengths and pairs of objects with masses or some other target physical magnitude) is so extrinsic, arbitrary and not physically natural – very much not the kind of thing we'd think should make up the metaphysical fundamentalia of the universe.

Defensively, the nominalist might argue that the difference isn't stark. They might note, as Field does, that standard platonistic regimentations of physical theories are also stated in terms of unattractively extrinsic relations[7]. So (one might claim) this point can't be used to argue that nominalistic logical regimentations of theories produced using the above method are explanatorily worse than platonistic regimentations. But it does seem plausible that the four place relation is even more arbitrary.

Thus, a nominalist who addresses the classic Quinean indispensability argument via the tricks I've proposed above, arguably faces a grounding problem. If the 4 place relation we used to paraphrase mass facts isn't metaphysically fundamental, what kind of metaphysically fundamental objects and relations *do* ground the fact that m_1 is π times more massive than m_2 ? Platonists can say that mass facts are grounded in a relation between physical objects and mathematical ones, e.g., the 3 place relation which holds between pairs of objects and their real-valued mass-ratio. But what can the nominalist say?

For example, a nominalist answering this challenge might draw on Sider's point that we should distinguish analysis of our actual language from analysis of metaphysical fundamental[13]. For example, we don't know what the most physically fundamental objects and properties are (maybe the entities in string theory, loop quantum gravity[19], or maybe there's a lower level of grounding). So it's plausible that we know rather little about what's metaphysically fundamental (but that's no barrier to logically regimenting our best scientific theory).

Given this distinction between logically regimenting our best theory grounding that theory in fundamentalia and the possibility of grounding facts using infinitely many relations, the nominalist might propose the following picture. Mass-ratio facts are grounded in facts about infinitely many mass properties[20] (or mass-ratio relations)[21]. However, the only way for finite creatures like us to talk about what mass properties objects have (since we cannot learn a language with a separate atomic predicate for each mass property) is by appealing to less intrinsic facts – like counterfactuals about idealized measurements. One could then think of the 4 place relation above as the results of an idealized experiment relating mass ratios to length ratios (e.g., the ratio of the lengths the objects would travel up a frictionless incline after receiving an identical impulse).

4.2 Reference Indispensability

Second, one might worry that the above paraphrases of our best scientific theory leave one with a problem accounting for our apparent ability to make certain claims which are not part of our best total scientific theory. For it seems that I can meaningfully state theories which *don't* imply that length is richly instantiated, e.g., some speculative physical theories which quantize space, or

[19]Thanks to REDACTED for all specific physics examples not cited.

[20]So, for example they might say there is one metaphysically fundamental mass property P which applies to an object iff it has mass 5 grams, one that applies if it has mass exactly 2, one that applies iff it has mass exactly π grams etc. And if one wants mass facts to be less intrinsic one could instead appeal to continuum many mass ratio relations.)

[21]See Hellman in [9] §3.4 for an example of this kind of story about infinitary grounding.

non-holistic theories which don't build in any assumption about the nature of space. But the nominalization tricks I've suggested above don't suffice - or don't clearly suffice- to do this.

It's prima facie unclear whether philosophers who don't take our language to include something like a platonistic three place relation 'x stands in mass ratio r to y' can make sense of our ability to state such *partial and/or alternative scientific theories*. Thus, it can seem like we are forced to accept the existence of mathematical objects in order to make sense of our ability to literally state certain, seemingly meaningful, scientific theories other than our current best theory.

Couldn't there be worlds where length isn't richly instantiated (and indeed we don't have any version of the measurement theoretic uniqueness result for space) but we do have determinate values for physical magnitudes such that the paraphrase strategy suggested above fails?

Whether or not such worlds are genuinely possible is debatable[22]. But for my purposes in this paper it suffices to note that this remaining worry (for a nominalist who uses the 'cheap tricks' above to logically regiment physical magnitude statements when formulating their total best theory), concerns accounting for paradigamatically philosophical rather than scientific phenomena. It appears that mathematical objects might be indispensable in accounting for certain metaphysical and otherwise philosophical intuitions (about the meta-

[22]For example, one might argue that objects in a given possible worlds can't stand in determinate mass ratio unless there's some result of an ideal experiment performed in that world which would relate this mass ratio to a ratio of lengths ('what would it mean to claim an object had a certain length, if that value couldn't be observed even indirectly?'). In this way one might argue that we shouldn't take intuitions about the possibility of about physical magnitudes taking on determinate values in an otherwise arbitrarily sparse universe (of the kind that, e.g., Eddon can be read as appealing to and which would directly motivate the existence of such worlds) at face value. However, this assumption that some possible idealized experiment could justify assigning a mass ratio by producing results that pair it with a corresponding length ratio doesn't suffice to ensure the *current/actual* existence of enough paths with lengths/pairs of points to ensure that the 4 place relation strategy proposed above pins down this mass ratio.

physical possibility of certain worlds and our ability to use language to draw certain distinctions between them) – not that they might be needed to account for scientific theorizing and explanation in any straightforward sense.

I have (speaking somewhat loosely) called this a referential indispensability worry, because it concerns our ability to 'refer' to certain sets of supposedly possible worlds (including ones where length isn't richly instantiated and yet length/mass etc. have certain determinate values) by uttering sentences which are true at exactly these worlds.

5 Conclusion

In this paper I have argued that we can plausibly answer classic Quinean indispensability worries driven by worries about the role of physical magnitude statements in our best scientific theories of the world. I have suggested physical magnitude statements instead pose a reference and grounding indispensability problem. Although not needed to state our best scientific theories, mathematical objects may be indispensable to accommodate certain philosophical intuitions about reference and grounding. If there are no numbers, how are humans able to finitely learn languages which draw certain distinctions between metaphysically possible worlds quite different from our own (not needed to state our best theory), and what could ground the truth of fundamental physical magnitude facts in the worlds?

If one accepts this reformulation of access worries, there are some important upshots for readers of different stripes.

First, hardcore naturalists may be inclined stop taking indispensability worries (based on concerns about physical magnitude statements) seriously. For we see that the nominalist's real problem doesn't concern stating or (in a sense) attractively explaining *scientific* facts involving mass and charge but rather accounting for certain a priori philosophical intuitions about metaphysical possibility, reference and grounding. Philosophical explanation is the sticking point, not scientific explanation.

Second, if we slot in the grounding and reference indispensability arguments into the philosophical literature in place of the classic Quinean indispensability argument above, but take the latter seriously (accepting the existence of mathematical objects on these grounds) there are several interesting consequences.

For one thing, if one accepts the existence of mathematical objects because of the above grounding challenge, then one has an automatic answer to certain access worries. I have in mind the suggestion [10] that if there hadn't been mathematical objects everything would have been the same. For, if masses are grounded in (and thus, plausibly, something like partly constituted by) a certain relation holding between physical objects and numbers, then the following (opposite) counterfactual intuition seems plausible: if numbers suddenly stopped existing then objects wouldn't have had masses, just as if hair suddenly stopped existing then people would stop having beards.

For another thing, consider arguments that we're only justified in believing mathematical objects exist necessarily because our only reason for accepting their existence in the first place is the role they play in our best scientific theories (as per the Quinean indispensability argument) [4]. The reference and grounding indispensability arguments raised in §4 present a twist on the classic Quinean indispensabillity argument which (if compelling) does justify the necessary existence of mathematical objects. In order to resolve the grounding and reference problems raised above, mathematical objects would need to exist necessarily.[23]

[23] According to the grounding indispensability argument, grounding facts about mass ratios in a three place relation between a pair of objects and number best explains how objects stand in determinate mass ratios at remote worlds where neither length nor mass is richly instantiated. According to the reference argument, interpreting our talk of mass ratios in terms of a relationship between objects and numbers explains how our claims that objects

A Comparison of Nominalist Paraphrase Strategies

Note that Field needs to use an infinite schema to state the archimedean axiom (making his nominalist paraphrases not finitely stateable). Also (since he thinks math is literally false) he gives a conservativity proof to show it's OK to use it in the sciences, but concerns have been raised about whether he makes unacceptable use of mathematics in giving that proof[3].

My preferred form of nominalization of physical statements follows (Berry's simplification [2] of) Hellman instead. In [9] Hellman advocates modal ifthenist paraphrases pure mathematical statements as (very roughy) making claims about what would logically necessarily have be true if there were (objects satisfying second order axioms describing) the numbers, functions etc. (while certain relations relate the same nominalistic objects they do in the actual world). Hellman is not a fictionalist like Field but) a traditional nominalist, who thinks mathematical claims express important truths (that just happen to be more illuminatingly formulated from Putnam's modal rather than ontological perspective) and provides suitable nominalistic logical regimentations of these claims in [9]. Thus he has no more need than the platonist for a conservativity proof to explain why false claims can be harmlessly asserted. And because facts about numbers and functions from physical points to numbers clearly definably supervene (in via a definition that can be given in second order logic) on facts about what physical points there are,

However it should be noted that benefits come at the cost of extra ideology. Like Field, Hellman appeals to a primitive modal logical possibility operator as Field also does. But unlike Field, Hellman also employs second order quantifi-

have a certain determinate mass ratio can be true at such remote possible worlds. In each case, the explanatory role which mathematical objects are invoked to play directly requires that they exist necessarily (even in remote possible worlds).

cation to describe pure mathematical structures, quantification into the logical possibility operator, and an actuality operator which allows one talk about what would happen to objects in logical possible scenarios where the objects that admire one another are exactly those objects that admire one another in the actual world. And Berry's [2] proposes a simplification of this using a 'conditional logical possibility' operator which can do the work of all the ideology mentioned above.

My aim in this paper is not to argue that one should accept the tools that Berry and Hellman use to paraphrase pure mathematics (most controversially Hellman's second order quantifier and Berry's conditional logical possibility operator).

Rather I have tried to show two things. First, if a nominalist finds Berry and Hellman's story about pure mathematics acceptable then paraphrasing physical magnitude statements with extra expressive power poses no problem.

Second (more importantly), the specific 'sparse magnitudes' problem which Putnam and raises and Eddon presses regarding physical magnitude statement (that the facts we want to talk about don't even supervene on a finite collection of nominalistic facts) can be solved by the cheap tricks I propose above if one only cares about literally stating one's actual best scientific theory. Thus (as advertised) the real challenge which physical magnitude problems pose for the nominalist concerns addressing the philosophically driven grounding and reference indispensability challenge, not Quine's original indispensability argument.

References

[1] Frank Arntzenius and Cian Dorr. *Space, Time, and Stuff.* Oxford University Press, April 2014.

[2] Sharon Berry. Modal Structuralism Simplified. *Canadian Journal of Philosophy*, 48(2), 2018.

[3] John P. Burgess and Gideon Rosen. *A Subject with no Object*. Oxford University Press, 1997.

[4] Mark Colyvan. *The Indispensability of Mathematics*. Oxford University Press, 2001.

[5] Donald Davidson. Truth and Meaning. *Synthese*, 17(1):304–323, 1967. bibtex:davidson1967.

[6] M. Eddon. Quantitative Properties. *Philosophy Compass*, 8(7):633–645, 2013.

[7] Hartry Field. *Science Without Numbers: A Defense of Nominalism*. Princeton University Press, 1980.

[8] Hartry Field. *Realism, Mathematics & Modality*, volume 16. Basil Blackwell, 1988.

[9] Geoffrey Hellman. *Mathematics Without Numbers*. Oxford University Press, USA, 1994.

[10] Carrie Jenkins. *Grounding Concepts: An Empirical Basis for Arithmetical Knowledge*. Oxford University Press, 2008.

[11] Hilary Putnam. *Philosophy of Logic*. London: Allen & Unwin, 1971.

[12] Theodore Sider. A Crash Course on Measurement Theory.

[13] Theodore Sider. *Writing the Book of the World*. Oxford University Press, 2011.