Hamkins' Multiverse and Applied Mathematics

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Hamkins' Multiverse

In 'The Set-Theoretic Multiverse'[1], set theorist Joel Hamkins proposes that

- Multiple hierarchies of sets V ("set theoretic universes") exist, making up the Set Theoretic Multiverse.
- There's no *intended hierarchy of sets V* which contains 'all possible subsets' of the sets it contains
- Rather, for each universe V, there's a wider universe V[G] which adds sets to V, including a new subset G of some infinite set in V.[1]

[1] note: the multiverse satisfies various other closure conditions as well.

Agenda

Hamkins motivates his Multiverse Proposal by appeal to

- A certain way of thinking about a mathematical technique called forcing
- An analogy between his proposed change in attitudes to set theory and historical changes in attitudes to geometry.

In this talk I'll

- Note a way Hamkins' proposal seems much more radical than this shift in attitudes to geometry.
- Consider an explanatory indispensability worry which arises from it.
- Note how a modal twist on Hamkins multiverse might solve this problem.

This talk in two sentences

Explanatory indispensability worry: If Hamkins says there's no mathematically preferred notion of 'all possible subsets/ways of choosing'...

- how can he replace or dispense with mathematical explanations of scientific facts that seem to appeal to this notion?

Note:

- Unlike traditional indispensability arguments (Quine, Baker and Colyvan) this worry attacks *truth value* anti-realism about math, not nominalism.

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Subsection Agenda: Tiny Background on Forcing

Let's review very basic facts about

- Forcing arguments from a conventional point of view
- A feature of them which (may) suggest Hamkins' different point of view

Conventional POV on Forcing I

The total hierarchy of sets V already contains 'all possible subsets' of sets it contains. So you can't add any extra subsets. But

- All consistent first order theories have a countable model.
  - Indeed, if the true V satisfies ZFC, so does some transitive countable model M.
- Any such model M must be 'missing' some subsets[2]
  - in particular, for certain infinite partial orders $\mathbb{P}$ in M we can prove there's a $G \subset \mathbb{P}$ which is M-generic, a property that implies $G \notin M$.
- Forcing arguments prove facts about how these countable models can be extended by adding some of these missing subsets of $\mathbb{P}$.
  - e.g., one proves a relative consistency claim by showing that any countable transitive model M of ZFC could be expanded into a countable transitive model $M[G]$ of $ZFC + \neg CH$.

[2] It can't contain all subsets of any infinite set it contains, by Cantor's diagonal argument.

A Key Suggestive Fact $\Vdash$

We appeal to a forcing relation $\Vdash$ which connects facts about M to facts about $M[G]$.

We prove that $M[G] \vDash ZFC + T$ by first proving a fact about M, that $\Vdash ZFC + T$.

- note $\Vdash \psi$ is just a claim about sets in M.
- It's provable[3] that (if there's any such G)
  - $\Vdash \psi \Leftrightarrow$ for every such G, $M[G] \vDash \psi$.

So we can work in some hierarchy[4] V, and prove $\Vdash \phi$ facts, but see this as *implicitly* telling us about what an extended universe V[G] would be like.

[3] via arguments that don't depend on M being countable

[4] i.e., only take 'set' to apply to the sets in V

Hamkins' vs. Conventional View

Two ways of thinking about forcing:

- Conventional View: The intended hierarchy of sets V contains 'all possible' subsets of sets it contains:
  - So there can't be a V-generic 'missing subset' G, or $V[G]$
  - Forcing only tells us about how countable models (which therefore lack some subsets) inside the true V could be extended.
- Hamkins' Multiverse: For any set theoretic universe V we may consider, there's a V-generic 'missing subset' G and a generic extension $V[G]$.

Why make the latter bold proposal? Hamkins says three things

Motivation 1: Forcing Phenomenology

First, Hamkins appeals to the phenomenology of forcing:

> *With forcing, we seem to have discovered the existence of other mathematical universes... **new set-theoretic worlds, extending our previous universe***

- I take it equally eminent set theorists who reject the multiverse program disagree.
- But I'll take Hamkins' descriptions for granted in this talk.
- I won't appeal to my or your phenomenology.
  - Mine involves a lot of experiences of flipping back for forgotten definitions!

Motivation 2: Forcing Terminology

Hamkins also says that current terminology fits nicely with the Multiverse view.

> *In the earlier days of forcing, theorems usually had the form* $Con(ZFC + \phi) \rightarrow Con(ZFC + \psi)$ *...[In contrast,] contemporary work would state the theorem as: If* ϕ*, then there is a forcing extension that satisfies* ψ*.*

For it (arguably) seems to talk about how the whole hierarchy of sets could be extended.

Motivation 3: Geometry

Hamkins writes:

> *There is a very strong analogy between the multiverse view in set theory and the most commonly held views about the nature of geometry.*

Note: the point here can't be merely that we should regard variant set theories as legitimate math, studying something real.

- Conventional approaches to forcing already do this:
- studying countable models of ZF+X within an intended hierarchy of sets V is clearly legitimate mathematics studying something real!

Motivation 3: Geometry

Rather Hamkins proposes a specific *historical parallel* between a change in attitudes to geometry, and the change he advocates re: set theory.

Hamkins' Historical Stages

1. Assume a unique intended subject matter
2. Accept alternate axiom systems as being legitimate math, but describing mere "playful reinterpretations" and "toy models" inside a larger intended universe, like
   - interpretations where 'lines' are great circles on a sphere in Euclidean space
   - mere countable models of ZFC within the true V
3. Consider alternate axiom systems as "fully real and geometrical"/set theoretic

Motivation 3: Geometry

What's involved in accepting variant axiom systems as fully real and geometrical/set theoretic? Note: this seems to involve

- somehow *building up* a new way of thinking about them as true of something other than a toy model
  - e.g. working with alternate geometries and "developing intuitions about what it's like to live in" them[5].
- not *debunking* a supposed contrast between legitimate interpretations and mere toy models[6] (as classic logical positivists might have it).
  - e.g., learning no geometry was physically special in the way 'true' geometry was supposed to be.

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[5] *At first, these alternative geometries were considered as curiosities,... In time, however, geometers gained experience in the alternative geometries, developing intuitions about what it is like to live in them, and gradually they accepted the alternatives as geometrically meaningful. Today....alternative geometries... are regarded as fully real and geometrical."*

[6] intuitions about there being more vs. less intended *physical* interpretation for 'point' and 'line'

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A Disanalogy with Geometry

With this quick sketch of Hamkins' Multiverse and the motivations for it in mind I'll now (in this section)

- point out a **difference** between the change in attitudes to set theory Hamkins advocates and the change in attitudes to geometry he invokes
- which may raise an explanatory indispensability problem

Common Geometrical Pluralism

Common view: Although various geometrical axiom systems are equally legitimate and "geometrical"...there's still (approx.[7]) a *physically correct geometry* which

- constrains physical points and lines approx. as naive geometry was supposed to.
- takes over the explanatory work of naive geometry
  - e.g., explains why some configurations of lines never occur.
- determines a (mathematical-axiom choice independent) right answer to questions like, 'is the parallel postulate true?'

But Hamkins' Multiverse includes no analog to a physically preferred geometry, and (we'll see) is hard to supplement with such a thing.

[7] Maybe there's a spectrum of more and less intended interpretations of 'line' as a notion of physical geometry.

Hamkins' Multiverse proposal raises questions/may require a revisionary attitude towards applied math in the science in two ways

- one of which generates the Explantory Indispensability worry mentioned above.

A Hidden Degree of Freedom

First, suppose infinitely many coin flips were going to take place.

Naive POV: We could assert a definite and elegant physical hypothesis by saying

- **Independent Flips**: "These coin flips are random and independent; all (combinatorially) possible outcomes are physically possible."
  - i.e., For every set of coinflips x (in set theory with ur-elements[8]), it's physically possible that exactly the coinflips in x will turn up heads.

Multiverse POV: This sentence will express different claims depending on the background set theory V we're using.

- and might not ever say something precise, since it's not clear how to semantically latch on to a unique V in the multiverse

[8] I take all *physical objects* to be ur-elements (and presumably no sets non-sets). So we have ZFC+U, with U: 'There is a set of physical objects.'

A Hidden Degree of Freedom II

Also note: physical theories that imply some version of Independent Flips look less elegant and a priori attractive than they did on the naive view:

- When working in some $V[G]$ extending the relevant V, we might wonder 'how does physics control the coinflips to avoid outcome G?'
- Maybe it's just a brute physical law that it does, but in this case ...
- Theories that imply Independent Flips turn out to have an extra 'degree of freedom'
  - they say the physically possible outcomes correspond to 'all subsets' relative to some particular, not intrinsically special, V in the multiverse.

Combinatorial Explanations I

Next and perhaps more importantly, there's a question about how to revise mathematical explanations of physical facts, where (speaking very abstractly) we take

- set theoretic facts to reflect
- general combinatorial constraints on 'all possible ways of choosing'
- and thereby explain regularities in how physical properties relations relate physical objects.

Combinatorial Explanations II

Naive POV: Facts about set theory with ur-elements reflect combinatorial constraints on how any properties can apply to the ur-elements and can thereby explain physical regularities.

Consider an infinite physical map.

Simple combinatorial explanation:

- The countries on this map aren't three colourable[9]

can provide a good (lawlike, counterfactual supporting) explanation for this physical fact

- This map has never been three coloured.

Because V (with ur-elements) witnesses 'all possible ways of choosing' from physical objects, the non-existence of a 3 coloring function implies the map isn't 3 colored.

[9] i.e., There's no function which takes these countries to $\{0, 1, 2\}$ such that no two adjacent countries are assigned to the same number'

Combinatorial Explanations III

And perhaps we can have more mathematical explanations of physical facts with a more complex logical structure (e.g., $\forall X \exists Y$ rather than $\neg \exists X$ so to speak)

- Regions on the map can't be stably held because for every (set coding) a way of stationing defending troops satisfying [such-and-such condition], there's a (set coding) a way of stationing attacking troops that satisfies [so-and-so condition].

Naive POV on Combinatorial Explanations

We believe V contains a set of physical objects and 'all possible subsets' of sets it contains, hence for any property $\phi(w_1, \ldots w_n,)$ definable using

- any relations in our language (not just $\in$) in ϕ
- **any** objects $w_1 \ldots w_n$ as parameters (not just sets in V) and

V contains a set of exactly the physical objects that have this property. So we have:

Full Comprehension Schema[10] for V
$(\forall z \in V)(\forall w_1) \ldots (\forall w_n)(\exists y \in V)(\forall x)[x \in y \Leftrightarrow ((x \in z) \wedge \phi)]$

[10] I slightly abuse notation, for legibility, in writing $z \in V$

What's the Multiverse POV?

But on the Multiverse POV: there's an equally legitimate perspective V' extending V, from which V is missing out on

- a set X of natural numbers and hence *also* (if ZFC+U is to remain true)...
- some sets of physical objects (and functions from the physical objects to numbers etc)
  - e.g., for each 1-1 function f in V from the numbers to the countries, I must now have the set of countries in the image of X under f, f[X] which wasn't in V.

So we can't use facts about set theory to explain (or even predict) physical regularities as we used to. V can be seen to violate

- the intuitive **full comprehension principle** that there's a set of countries corresponding to each property ϕ definable with parameters above
  - as shown by considering the property of being $\in X$

Note re: Simple Combinatorial Explanations

If we took the universe V[G] to be a **traditional intended model of set theory** (i.e., to contain sets witnessing modal facts about 'all possible subsets' hence satisfy Full Comprehension and contain a three-coloring function if the map is actually three colored), we could justify inference from facts about V to non-three colorability as follows [11]

- going from V to a forcing extension V[G] won't change facts about whether there's a three coloring.
  - V[G] only adds generic sets and the property of being a three coloring for this map-adjacency coding set is one can one always avoid when progressively specifying which numbers are in a set

But the point is that *Hamkins* can't say that the V[G] we are working in (or *any* universe V in the multiverse) is a traditional intended model in this sense!

[11] assuming V contains a set coding the adjacency relations on the map

The Explanatory Indispensability Question

The question is:

- Accepting the multiverse forces us to reject the traditional picture connecting set theory to non-mathematical facts on which
  - set existence facts are supposed to reflect **lawlike modal constraints** on how any objects can be related by any relations
  - and therefore satisfy full comprehension.
- If we break this link how can (why do) facts about set theoretic universes explain anything about regularities in the physical world?

Note

This prima facie question remains

- even if we can show that *all* universes in the multiverse agree on the set theoretic sentence used in combinatorial explanation (e.g.,the claim that there's no three coloring function, or that for every set coding a defending troop distribution there's a set coding an attacking distribution)

Two Kinds of Multiverse Theorist I

On the mainstream point of view, there's favored notion of 'all possible ways of choosing' that's

- reflected in combinatorial constraints on how all physical properties relate all physical objects
- usable to describe the intended structure of the hierarchy of sets (up to width)
  - Biographical anecdote: I actually seem to remember modal talk being used to describe iterative hierarchy when I first met the sets in intro analysis
- I tend to think of this notion modally, as somewhat analogous to a primative modal notion of (non-humean) physical possibility

Two Kinds of Multiverse Theorist II

In principle, a platonic multiverse theorist can either

- Deny there's any uniquely favored notion of 'all possible ways of choosing' that we can latch on to (Hamkins??)
- Merely deny that a single hierarchy of sets does/can witness facts about all possible ways of choosing in the way traditionally expected (i.e., by containing sets corresponding to all possible ways of choosing from sets it contains). (Scambler?)
  - so facts about the multiverse as a whole reflect all possible ways of choosing

I'll discuss options of both kinds below.

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Op 1:A Physically Special V?

Hamkins could say there's a physically special V_p with ur-elements such that

- It's a physical law that V_p contains **all** 'sufficiently physically definable' subsets of sets it contains
  - i.e. physical law prevents properties from applying to actual physical objects in a way that isn't witnessed by the existence of an actual set of objects in V_p
- **(maybe)** V_p contains **only** the sets corresponding to physically possible outcomes of scenarios we'd describe as independnet random events.

Optional Note Re: Metaphysical Possibility

For Context: I take the following to be a natural way to think of the modal behavoir of any (ZFC+U satisfying extension of) a universe within the multiverse V_p

- the pure sets structure of V_p is the same in all metaphysically possible worlds
- V_p satisfies ZFC+U in all metaphysically possible worlds.
- if some plurality of objects form a set (in the sense of V_p) in some metaphysically possible scenario, then in *every* metaphysically possible scenario where all these objects exist they form a set (in the sense of V_p).
  - e.g., it's not the case that if different coinflips had come up heads then different sets of coins would exist

Op 1:A Physically Special V?

Note, as discussed above: when working in some $V_p[G]$ extending the relevant V_p, we might wonder 'how does physics control the outcomes of seemingly random independent events (e.g., coin tosses), to avoid 'realizing'[12] a missing subset? '

- Maybe it's just a brute physical law that it does, but...
- Is it plausible that a physical law steps in and stops the map from being three-colored in a way that corresponds to one of those missing functions[13]?

[12] i.e. letting us use physical vocabulary to define

[13] and does the same to prevent three scenting, three texturing etc

Op 1: Physically Special V

Other awkwardnesses

- Is the notion of 'suitably physically defined' properties too gerrymandered to figure in a legit physical law?
- Physical theories like QM turn out to have an extra 'degree of freedom'
  - they say the physically possible outcomes correspond to 'all possible subsets' relative to some particular, not intrinsically special, V_p in the multiverse.

Opt 2: Stipulate Comprehension

We could *stipulate* that by (our background) V, we always mean some universe which contains all sets definable from physical parameters[14]. But note

- Naive set theoretic explanations like, 'The map won't be three colored because it's not three colorable' invoked something with **counterfactual supporting law-like** force.
- On this multiverse approach, the non-existence of a three coloring function still implies that the map won't ever be three colored
- BUT we only have a dormative virtue non-explanation:
  - 'The map won't ever be three colored because a hierarchy of sets V that contains sets coding all ways physical properties will actually apply doesn't contain a three coloring function.'

[14] Though no V could contain sets witnessing all properties definable with objects from the set theoretic multiverse

Opt 3: FOL Surrogates

Say we can always replace these appeals to set theory/'all possible subsets' (like the example below) with appeals to first order logic.

- 'That map was never three colored, because it is not three colorable (i.e., there is no three coloring function.')'

Strat. 1: In some cases we could just directly appeal to FOL and say

- 'That map will never be three colored because some countries on it are related like [such-and-such][15], so by [FOL deduction] the map isn't three colored] .'
- Problems:
  - This only works when we happen to **know** the finitely many facts about the map that FOL entail it isn't three colorable.
  - And we don't need to know such facts to correctly hypothesize the above explanation.
  - This is specific FOL deduction is nowhere near as unifying as the naive explanation[16].

[15] By completeness and compactness there will always some finite collection of facts which imply that the map isn't three colored.

[16] C.f. appeals to the distinction between program vs. process explanations in

FOL Surrogates

Strat 2: To mimic this unificatory power, we might

- rewrite explanations to appeal to laws involving special principled facts about *derivability*[17] rather than (the rejected notion of) ‘all possible ways of choosing’ objects from a plurality.
  - e.g., “The map isn’t ever three colored because it’s FOL derivable from *some* true sentence about how finitely many countries are related by adjacency that it won’t be three colored’.”

[17] Forcing doesn’t change the natural numbers in V, hence won’t change facts about what your background set theory thinks is derivable.

FOL Surrogates

- BUT Hamkins' Multiverse doesn't just contain set theoretic universes that differ as per forcing but also ones that differ on well foundedness and intended models of the natural numbers.
  - So maybe appeal to principled laws involving derivability is also off the table?
- And I'm not sure how far this can be extended to handle more complex explanations as above.

Op 4: Invoke Whole Multiverse

Finally you could replace appeal to any V with appeal to the whole multiverse.

- i.e.,‘The map isn’t three colorable’ = there’s no V with ur elements in the multiverse which contains a three coloring function for it.
- But then how can we quantify over all sets anywhere in the multiverse when doing science but not when doing pure math?

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A Modal Twist to the Multiverse

Modal Multiverse (MM) I

I'll now suggest an alternative approach to Hamkins' multiverse

The **Modal (Hamkins') Multiverse View** accepts the naive notion of 'all possible subsets' and hence

- A unique intended hierarchy of sets V (up to width) containing 'all possible subsets' of the sets it contains. etc.

However, it understands forcing unconventionally,

- in a way suggested by Hamkins' remarks about geometry

Inspiration from Hamkins on Geometry

Hamkins describes geometers 'building up' to accepting various axiom systems as geometrically meaningful via

> *gain[ing] experience in ...alternate geometries, developing intuitions about what it is like to live in them,*

Idea: Maybe coming to find alternative axioms 'geometrically meaningful' rather than merely true under a "playful reinterpretation"means something like:

- coming to think of them as describing *points* and *lines* (in something like) a conceivable physical space?
- i.e., becoming able to concieve of a scenario where these axioms as expressing truths while keeping approx. their naive physical implications

Inspiration from Hamkins on Geometry

In this case we might say

Different geometries reveal facts about

- (physically impossible) scenarios where physical space has a radically different structure.

and, remembering analogous applications of set theory

Different set theories reveal facts about

- metaphysically impossible scenarios[18] where the facts about logical possibility are different, so the true V containing 'all (logically) possible subsets' satisfies different axioms.

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[18] See [3]

Modal Multiverse II

Modal Hamkins' Multiverse View: forcing extensions study what would be true in metaphysically impossible scenarios where

- there are more logically possible ways for predicates to apply
- so that[19] *our* 'full width' hierarchy of sets V exists within V[G], the wider intended hierarchy of sets for in this scenario
- but the laws of FOL still hold[20] so we can still infer syntactic consistency from truth.

These (rather psychedelic) scenarios are like ones where 3 or 5, not 4, different sundaes are buildable in a sundae bar with two toppings[21]

[19] an intrinsic duplicate of

[20] and no contradiction is true

[21] But they only change how it's logically possible for a predicate to apply within an infinite collection.

What's the 'Fully Grown-up Universe'?

For Hamkins criticises conventional approaches to forcing by saying

> *"[This] toy model perspective can ultimately be unsatisfying... since it is of course in each case not the toy model in which we are interested, but rather the fully grown-up universe."*

But can the Classic Multiverse theorist make sense of this distinction?

- Yes they say forcing tells us about a V[G] genuinely extending our current V, not a countable model M inside it.
- But (Hamkins says [1]), for each universe V there's a V' in the multiverse from whose persepective V is countable.[22]
- So it seems V[G] is also a 'toy model' in whatever sense M is.

[22] i.e. V' extends V and contains a function pairing it 1-1 with ω in V'

Modal Multiverse on ‘Fully Grown-up Universes’

In contrast, the Modal Multiverse theorist can make crisp sense of Hamkins’ claim to study “fully grown-up universe[s]" that are

- more "interest[ing]" (and ‘genuinely set theoretic’) than facts about countable models of ZFC.
- because they keep our ZFC transcendent notion of all possible subsets

The MM theorist can say forcing arguments reveal facts about

- scenarios where a structure satisfying our naive conception of the V - as capturing ‘all possible ways of choosing subsets’ - satisfies different axioms.

Objection to MM

Admitted weakness of MM: the modal reading of forcing theorems doesn't support inference to syntactic consistency claims $Con(ZFC + \phi) \rightarrow Con(ZFC + \psi)$,

- For truth in some metaphysically impossible scenario doesn't imply consistency.

Answer

- Even if we accept Hamkins subtler and more controversial arguments we'd probably want to say
  - the (best) justification for consistency claims comes from conventional reasoning about countable models.
- And once we know this point about countable models, we can infer consistency from truth in a metaphysically impossible forcing extension of the true V...

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Conclusion I

I've argued we can equally (or better) honor Hamkins' motivations

- forcing notation and (Hamkins') phenomenology
- analogy with geometry

by adopting a modal version of Hamkins' multiverse view.

Modal Multiverse: There's an intended hierarchy of sets V (up to width), but forcing extensions reveal metaphysically impossible scenarios where

- more things are logically possible so (a copy) of our intended V can exist inside a fatter V[G] which captures 'all logically possible ways of choosing' subsets *in this scenario*.

Conclusion II

On the Modal Hamkins' Multiverse view, just as

- studying different geometries illuminates certain (physically impossible) scenarios where the laws of physical space are different
- studying different set theories illuminates certain (metaphysically impossible) scenarios where the laws of logical possibility are different

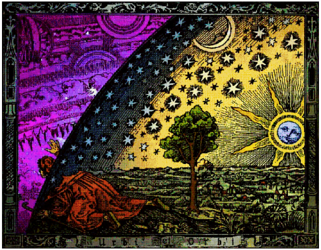
Switching to to this view lets us avoid problems about how to

- dispense with physical explanations invoking the notion of 'all possible subsets'
- make sense of Hamkins' remarks about interest in 'full-grown set theoretic universes' vs. mere toy models.

Conclusion III

Note: I'm not advocating the Modal (Hamkins-type) Multiverse View myself!

- I'm a fan of a conventional approach to the width of the iterative hierarchy of sets (with potentialism just about height)
- But maybe considering it helps bring out the interest and potential radicalness and strangeness of philosophical questions raised by Hamkins' project



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Can't Mirror Solution to App. Problem for Geometry I

Naive POV: facts about set theory constrains the physical world via the this principle: V contains 'all possible subsets', in a way that makes it metaphysically impossible to pick out a missing subset of a set z in V

- using **any** objects as parameters (not just sets in V) and any relations in our language (not just $\in$)
- any variant we might speak in contexts where we add new predicates or names **or drop quantifier restrictions**[2]

So we accept the following schema in all contexts where the range of our quantifiers includes V:

Necessary Full Comprehension Schema[23] for V

$$\Box(\forall z \in V)(\forall w_1)\ldots(\forall w_n)(\exists y \in V)(\forall x)[x \in y \Leftrightarrow ((x \in z) \wedge \phi)]$$

- and $\Box$ expresses metaphysical necessity.

.

[23] I slightly abuse notation, for legibility, in writing $z \in V$

An Objection re: Mathematical Parity

Objection: How can the Modal Multiverse theorist say variant axioms systems are "genuinely set theoretic" (in the relevant sense) if there's one true V, and facts about this reflect the answer to naive questions about CH?

Response:

First, note: We say variant geometries are all 'genuinely geometrical' despite the fact that some

- fit the intended applications of naive geometry better
- are more relevant to naive geometers' questions about the parallel postulate
- are specially mathematically interesting or useful in various ways

Extending Analogy to Motivations for Pluralism

Also the MM theorist can say her motivations for saying different set theoretic axioms reflect something genuinely set theoretic neatly parallel her reactions to geometry

- Learning that
  - a priori reason couldn't settle central questions about physical geometry (like the parallel postulate),
  - we can rigorously work with and (kinda) imagine physical possibilities corresponding to variant axioms

  motivates taking multiple options to be legit topics for mathematical geometry.
- Learning via Forcing arguments that
  - a priori reason can't settle certain basic points (like CH) about the intended V
  - but we can rigorously study a range of epistemic possibilities for the V reflecting true logical possibility facts.

  motivates taking these metaphysically impossible scenarios to be equally legit topics of mathematical set theory.

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