The Nominalist's Real Problem with Physical Magnitude Statements

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Abstract

In this paper I'll argue that mathematical nominalists who accept a notion of logical possibility can plausibly answer Quinean and Explanatory Indispensability worries concerning physical magnitude statements by means of two formal cheap tricks. However, important worries about reference and grounding remain.

1 Introduction

Indispensability arguments maintain that (in one way or another) we cannot adequately make sense of our current scientific knowledge without accepting the existence of mathematical objects. For example, Quine's classic Indispensability argument holds that we need to quantify over mathematical objects to *literally state* our best scientific theories, and this commits us to the existence of such objects. And Baker's Explanatory Indispensability argument[2] points out that mathematical facts do the heavy lifting in certain scientific explanations and maintains that mathematical objects are needed to *best explain* certain scientific facts.

In this paper, I'll argue that we can plausibly answer both Quinean and Explanatory indispensability arguments, if we accept a certain independently-motivated modal notion from the literature on potentialist set theory. In 2 I'll introduce the key modal notion I want to use (a kind of logical possibility) and motivate the idea that nominalistic mathematical explanations for scientific facts employing this notion can equal (and perhaps even improve on) the explanatory power and virtue of corresponding Platonist theories.

Then, I'll provide a general nominalistic paraphrase strategy and argue that it can be used to answer the pair of indispensability arguments above. Specifically, I'll argue that it lets us address two key issues in the indispensability literature: a worry that nominalistic paraphrases of scientific theories aren't sufficiently general or explanatory, and a concern (going back to [19] but refined in the subsequent literature [9, 12, 8]) about physical theories that make claims about physical magnitudes like length, mass, temperature etc.

In §3 I'll present a basic nominalization strategy (along the lines of modal if-thenist proposals in [12, 3]) and note some advantages of this approach. In §4 I'll review the special problems associated with nominalistically paraphrasing theories involving physical magnitudes, and argue that adding two formal 'cheap tricks' to the basic if-thenist modal paraphrase strategy lets us solve these problems.

Before beginning, however, I should note that the paraphrases I advocate won't be helpful to every nominalist. Philosophers who reject mathematical objects as part of a general physicalist project may reject the key notion of logical possibility used in my paraphrases as insufficiently physical, or take issue with the substantivalsm about space which my proposal (like Field's in [9]) assumes. However, another reason for rejecting mathematical objects comes from considerations favoring a modal approach to mathematics. As Putnam noted in [18], in many contexts it seems we can equally well take either a modal or a Platonistic perspective on pure mathematics. However certain puzzles (concerning the Burali-Forti paradox and the height of the hierarchy of sets) appear to favor a modal approach to pure higher set theory [12, 15]. In this paper I aim to clarify whether Quine's or Baker's indispensability arguments block taking a similarly modal perspective on mathematics as a whole.

2 Background And Motivation

2.1 Quinean and Explanatory Indispensability Arguments

As noted above, Quine proposed a classic indispensability argument along the following lines. We can't state our best scientific theories without quantifying over mathematical objects. And we should believe in the objects quantified over by our best scientific theories. So we should believe in mathematical objects.

Some nominalists, like Hartry Field, have answered this challenge head on, by rewriting scientific theories to avoid quantification over mathematical objects[9]. Others have rejected this demand for literal statement [5, 1, 23]. Drawing on scientists' use of idealized models and known falsehoods, like talk of infinitely deep oceans or continuous population functions, they say that it's OK if we can only evoke the scientific claims we believe are true by engaging in a fiction/pretense and saying things that are (literally) false. Accordingly, we can unproblematically quantify over mathematical objects in communicating our best scientific theories, even though no mathematical objects exist.

The Explanatory Indispensability argument strikes back at both kinds of nominalists: suggesting that mathematical objects are needed to **best explain** the phenomena accounted for in our scientific theories. Even if we don't need to believe in all objects quantified over in communicating our best scientific theories, the existence of mathematical objects is required to best explain the facts these theories explain. And even when we can literally state a nominalist theory which entails all the same consequences for non-mathematical objects as per Field, the resulting nominalist theory need not — and sometimes does not — provide an equally good explanation for these phenomena. So we have an inference to the best explanation for the existence of mathematical objects. As the Stanford Encyclopedia of Philosophy puts it it [7]:

> [the] poster child for [arguments for the] explanatory indispensibility of mathematical objects is Baker's Magicadas explanation.
>
> North American Magicadas are found to have life cycles of 13 or 17 years. It is proposed by some biologists that there is an evolutionary advantage in having such prime-numbered life cycles. Prime-numbered life cycles mean that the Magicadas avoid competition, potential predators, and hybridisation. The idea is quite simple: because prime numbers have no non-trivial factors, there are very few other life cycles that can be synchronised with a prime-numbered life cycle. The Magicadas thus have an effective avoidance strategy that, under certain conditions, will be selected for. While the explanation being advanced involves biology (e.g., evolutionary theory, theories of competition and predation), a crucial part of the explanation comes from number theory, namely, the fundamental fact about prime numbers.

In this paper, I'll argue that applying certain formal 'cheap tricks' to the kind of modal if-thenist paraphrases discussed in (c.f. Hellman in [12]) lets us us address classic Quinean indispensability worries concerning paraphrasing scientific theories that appeal to physical magnitudes like length and charge. I will also argue that the relevant modal if-thenist paraphrases (where defined) are explanatoraly at least as good as, and arguably better than, corresponding Platonist explanations.

2.2 Conditional Logical Possibility

For simplicity, I will formulate my paraphrases using the notion of conditional logical possibility from the streamlined formulation of potentialist set theory in [3]. However, it should be noted that we could do the same formal work using other notions favored by different developers of potentialist set theory (e.g., by using Hellman's combination of a second order function and relation quantifier and actuality operator employed by [13] as [3] makes clear[1]).

When considering logical possibility simpliciter[2], we ask whether there is any way of choosing a domain and extensions for relations in that domain which makes a certain claim true. When evaluating conditional logical possibility $\Diamond_{R_1 \ldots R_n}$ we do almost the same, but we also hold fixed the application of certain specific relations $R_1 \ldots R_n$.

I will use the notation $\Diamond_{R_1 \ldots R_n}$ to express claims about what's logically possible *given the facts about how certain relations apply.* Consider:

> **C&B**: 'It is logically impossible, given what cats and baskets there are, that each cat is sleeping in a basket and no two cats are sleeping in the same basket.'

There's an intuitive sense of 'logically impossible' on which this claim will be true iff *there are more cats than baskets* in the actual world. I'd write this as follows.

$\neg\Diamond_{cat,basket}$ [Each cat is sleeping in a basket and no two cats are sleeping on the same basket.]

Using this language of conditional logical possibility, we can express the

[1] We could probably also do the same work by coding all nominalisticly acceptable objects and relations satisfying the definable supervenience conditions below with sets and then using the notions of interpretational possibility, the logically necessary essences of sets, and plural logic Linnebo uses to develop potentialist set theory in [16].

[2] See [4] on reasons to accept a primitive modal logical possibility operator and see [10, 12] on the appeal of understanding mathematics, including applied mathematics, in terms of logical possibility rather than mathematical objects.

non-three colourability claim above follows[3] :

$\neg\Diamond_{adjacent,country}$ Each country is either yellow, green or blue and no two adjacent countries are the same color.

2.3 Motivating Case: Three Colourability

To illustrate how conditional logical possibility claims are useful for nominalizing mathematical explanations of physical phenomena (as needed to answer Explanatory Indispensability worries) – and why one might think these explanations actually *improve on* Platonistic ones – let's think more about the three colourability claims above.

Suppose that a certain map (perhaps one with infinitely many countries) has never actually been three colored. A good explanation for this fact might be that (in a mathematical sense) the map isn't three colorable.

A natural Platonistic explanation along these lines goes as follows.

Platonistic Non-Three-Colorability: There is no function (in

[3]One can further explain and motivate my notion of conditional logical possibility by appeal to Stuart Shapiro's notion of structures in works like [21]. Shapiro introduces a notion of systems, consisting of some objects to which some relations $R_1 ... R_n$ apply, considered under some relations e.g., "An extended family is a system of people with blood and marital relationships, a chess configuration is a system of pieces under spatial and "possible move" relationships, a symphony is a system of tones under temporal and harmonic relationships, and a baseball defense is a collection of people with on-field spatial and "defensive-role" relations."

Then he says that a structures are 'the abstract form' of a system, which we get by "highlighting the interrelationships among the objects, and ignoring any features of them that do not affect how they relate to other objects in the system." Thus, for example, the natural-number structure will be exemplified by a number of different systems: the strings on a finite alphabet in lexical order, an infinite sequence of strokes, an infinite sequence of distinct moments of time, etc'

In these terms we might say that my notion of logical possibility given the facts about how certain relations apply is logical possibility given the fact about what structure is instantiated by the objects satisfying at least one of these relations (considered under these relations). That is:

It is logical possible, given the $R_1 \dots R_n$ facts, that ϕ (i.e., $\Diamond_{R_1 \dots R_n}$ iff some logically possible scenario makes ϕ true while holding fixed what structure (in Shapiro's sense) the system formed by the objects related by $R_1 \dots R_n$ (considered under the relations $R_1 \dots R_n$) instantiates.

the sense of a set of ordered pairs) which takes countries on the map to numbers $\{1, 2, 3\}$ in a such a way that adjacent countries are always taken to distinct numbers.

However, we now have an additional nominalist version of this claim to consider.

Modal Non-Three-Colourability: $\neg\Diamond_{adjacent,country}$ Each country is either yellow, green or blue and no two adjacent countries are the same color.

And the above modal explanation can seem to be at least as good, indeed better than the nominalist explanation.

In particular, one might argue that the Platonistic non-three-colourability principle only intuitively explains because we have background knowledge of a relationship between set theoretic facts and the modal facts above. Specifically, we think that that there are functions corresponding to *all possible ways* of pairing countries with one of the numbers 1, 2 or 3, and hence all possible ways of 'choosing' how to color these countries. For if we didn't accept this then we would have no reason to suppose that there really was a function corresponding to a potential 3-coloring.

Thus, it may seem that the real explanatory work here is being done by the modal principle: claims about what mathematical objects (e.g. set theoretic functions exist witnessing facts about how it would be logically possible for any predicates to apply) exist don't really add anything to the explanation[4].

I hope the above toy explanation provides a nice motivating example of the kind of thing that I'll be trying to do: provide a nominalistic-mathematical

[4]See REDACTED for more argument on this point.

explanations for scientific facts which is as good as (and in some senses even intuitively better than) Platonistic ones. Interestingly, the modal nature of the nominalist paraphrase arguably matches ordinary language better than Platonistic paraphrases do. We tend to express the above thought modally, by talking about maps being three color*able*, rather than ontologically, by talking about maps *having three colourings*.

3 Nominalistic Paraphrase Strategy

Now let's turn to the task of providing a general paraphrase strategy which addresses the point above. I'll suggest a paraphrase strategy T by which any Platonist sentence ϕ satisfying a certain definable supervenience condition can be turned into a nominalisticly acceptable sentence $T(\phi)$ which has the same non-mathematical content.

The basic idea behind my proposal is a familiar modal twist on if-thenism, which has been developed in the case of pure mathematics by Putnam and Hellman[13, 18]. First, we come up some axioms completely pinning down (structural facts about) all the extra mathematical objects and relations the Platonist wants to appeal to. For example, in the case of the natural numbers, these axioms might be a version of the second order Peano Axioms characterizing the natural number structure (written using the conditional possibility operator[5]). Then, we nominalisticly formalize mathematicians' apparent claims that ϕ as really saying that if there were objects satisfying the axioms then ϕ would be true.[6]

[5] See Appendix Afor a demonstration of how to replace second order quantification with the conditional logical possibility operator.

[6] In cases where we have a categorical description of the relevant structure (i.e., any two structures satisfying the description would have to be isomorphic to each other), this gives bivalent truth conditions for all pure mathematical statements. Note that when it's necessary to use second order quantification to pin down a categorical conception of the relevant structure, we can do this purely in the language of conditional logical possibility. This is demonstrated in [3]. Also note that the potentialist position I'm considering advocates a separate 'potentialist'

Roughly speaking, the idea will be that $T(\phi)$ will assert (using conditional logical possibility) that ϕ would be true if we supplemented the world with the mathematical (and applied mathematical) objects which the Platonist was assuming existed when they asserted ϕ.

To apply this idea, we need to specify what mathematical objects (and relations involving them) the Platonist takes there to be in terms of facts about how some nominalistic relations (i.e., relations whose extension the Platonist and nominalist agree on) apply. Informally, I'll say that the application of some plurality of Platonistic relations $\mathcal{P}$ (used to describe the mathematical objects and applied mathematical facts) **definably supervenes** on that of some finite plurality of nominalistic relations $\mathcal{N}$ when the following holds (see appendix A for more technical details). Some sentence D (in the language of conditional logical possibility[7]), which the Platonist accepts as a metaphysically necessary truth, completely specifies how all the platonistic relations in $\mathcal{P}$ are supposed to apply at each metaphysically possible world (by specifying how the objects satisfying $\mathcal{P}$ are supposed to relate to each other and to the objects satisfying $\mathcal{N}$ at that world). For example, a definable supervenience description D will usually include a categorical description of all relevant pure mathematical structures (like the natural numbers under successor). Note that this sentence D, being finite, can only contain finitely many nominalistic relations.

When some Platonistic vocabulary $\mathcal{P}$ definably supervenes on some nominalistic vocabulary $\mathcal{N}$, we can nominalistically translate every sentence ϕ which only employs relations in $\mathcal{P}, \mathcal{N}$ (and has all quantifiers restricted to objects related by one of the relations in $\mathcal{P}, \mathcal{N}$[8]). For the truth value of all such sentences ϕ will be completely determined by the structure of objects satisfying

(but still modal) treatment for unrestricted set theory.

[7]That is, some sentence in the language of first order logic supplemented with the conditional logical possibility operator.

[8]More formally, those objects which take part in some tuple satisfying one of these relations.

the Platonistic and nominalistic relations $\mathcal{P}, \mathcal{N}$. And one can use the relevant definable supervenience description D to precisely pin down total this platonistic structure (at each possible world) in terms of the intended relationship between platonistic objects and relations and nominalistically acceptable ones. Note that, as one can categorically specify standard mathematical structures using conditional logical possibility[9], such structures automatically satisfy the definable supervenience condition.

So, for example, suppose that we want to translate a Platonist sentence that quantifies over numbers, pigs and dogs. Then, a supervenient description D for the Platonist relations in this sentence over the nominalist ones might specify that there are numbers satisfying (a modal version of) the second order Peano Axioms — and claim that no number is either a pig or a dog and the claim that 'successor' only relates numbers. And if we want to translate claims about a (supposed) layer of sets of goats, a definable supervenience condition D for the relevant notions of 'set' and 'element' will include a statement which uniquely pins down what sets of goats the Platonists takes to exist, in terms of what goats exist (in a sense to be clarified below). For example, in this case D will need to express something like the idea that there's a set of goats corresponding to each possible 'way of choosing' some of the goats.

Thus, when the (the application of) a sentence's platonist vocabulary definably supervenes on (the application of) its nominalistic vocabulary as defined above, we can use the conditional logical possibility operator to write an if-thenist paraphrase of ϕ as follows. If $\mathcal{N}$ is a list of all nominalistic vocabulary used in D and ϕ then we have the following:

$T(\phi) = \Box_{\mathcal{N}}(D \to \phi)$

Intuitively, this says that it's logically necessary, given the structure of objects satisfying the nominalistic relations $\mathcal{N}$, that *if* there were (objects with

[9]See appendix A and [3]

the intended structure of the) relevant mathematical objects then ϕ would be true. Note that the Platonist must believe it is always logically possible to supplement the non-mathematical objects at each possible world with additional objects so that D is satisfied, for the Platonist thinks that D is a metaphysically necessary truth.

To see how this plays out concretely, consider the statement GOATS 'There are are some goats who admire only each other.' Applying the above paraphrase strategy gives us a sentence T(GOATS) as follows.

$\Box_{goat,admire}$[There are (objects with the intended structure of) the sets of goats $\rightarrow$ There is a set of goats x, such that the goats in x admire only each other.]

This nominalistic paraphrase strategy is good in the following sense. From a nominalist point of view, $T(\phi)$ captures all the non-mathematical content that the Platonist *intended* to express by ϕ. Where it is defined, $T(\phi)$ is true at exactly those metaphysically possible worlds where the Platonist thinks ϕ is true. To put this point another way, if we suppose the Platonist assumptions articulated in the relevant definable supervenience D are metaphysically necessary truths (as the Platonist believes), then it will be metaphysically necessary that ϕ is true if and only if $T(\phi)$.

Finally, a nominalist might worry about the above translations' use of mathematical vocabulary inside the $\Diamond/\Box$ of logical possibility/necessity. As stated, they talk about how it would be logically (not to say metaphysically!) possible for there to objects like sets with ur-elements. If you are a nominalist who thinks that 'set' is a meaningful predicate which just happens to necessarily not apply to anything (as Kripke argued for 'unicorn'), this is fine. However, those who don't like this option should note that we could use any other first order predicates and relations that don't occur in $\mathscr{N}$ instead. For example, we could uniformly replace ‘set’ and ‘element’ in the translation above with,‘angel’ and ‘...is transubstantiated into...’ in our $T(\phi)$. This strategy is reminiscent of Putnam’s strategy for stating potentialist set theory in [18].

3.1 Advantages

Note that this basic if-thenist paraphrase strategy has no problem applying to Platonist theories that quantify over arbitrarily large mathematical structures (provided we have a suitable description of them). For it is logically possible that existing physical structures exist alongside arbitrarily large mathematical structures. This provides an advantage over nominalization strategies like Rizza’s in [20]. For, the latter strategies require us to find a copy of whatever mathematical structures the platonist theory to be paraphrased quantifies over in the physical world. This limitation on size prevents these strategies from capturing the unifying explanatory power of platonist mathematical explanations that appeal to very large mathematical structures.

Also, my modal if-thenist paraphrase strategy always produces finitely stateable theories where it applies. In §4 I will argue that my strategy can be applied to solve the problems which drove Field to provide infinitary nominalistic paraphrases for platonist theories (stateable via a schema)[10].

[10] However certain disadvantages may also be admitted. Most obviously, accepting the conditional logical possibility operator is controversial (though, recall, Field himself advocates accepting a primitive logical possibility operator and uses it in his argument for conservatism). Also the kind of paraphrases of physical magnitude statements provided will not be as attractively ‘intrinsic’ in the way Field wants.

4 Physical Magnitude Statements

4.1 The Problem

Now how far does the nominalistic paraphrase strategy I've proposed generalize? For example, can we use it to nominalize all the mathematical explanations for scientific facts that have been used to make Explanatory indispensability arguments? If we look at the nice list of such explanations provided by [17] the following picture emerges. The basic modal if-thenist paraphrase strategy stated so far can be immediately applied to about half the cases Lyon mentions. For example, it can be used to nominalistically explain the fact that no walk ever crosses each Köningsburg bridge exactly once – and the same goes every constellation of more than two islands each of which sports an odd number of bridges.

However, it's not clear that this basic paraphrase strategy can be used to nominalize the other half of the explanations on Lyon's list: the mathematical explanations of physical facts involving distance and other physical magnitudes (for example, Lyon lists an explanation for the hexagonal shape of honeycombs which appeals to the fact that this shape optimizes the ratio of area to perimeter).

The problem in these cases is this. As we saw in §3, the basic modal if-thenist paraphrase strategy is defined when, and only when, all notions used in the Platonist theory being paraphrased definably supervene on the facts about finitely many nominalistic relations apply. However, as dramatized by Putnam's counting argument in [19] and the following literature, it's not clear that this is true of Platonist theories involving physical magnitudes like mass, charge and length. The Platonist can state their theory using notions like a mass relation which relates physical objects to their mass in grams, or a mass ratio relation which relates pairs of objects to a number that's the ratio between their masses.

It's not clear that we can specify suitable definable supervenience conditions for these notions.

As noted by philosophers like Field in [9], appeal to measurement theoretic uniqueness theorems suggests an answer to this problem (as regards length). For when certain assumptions (which I'll call the claim that space is richly instantiated[11]) hold, we can give a definite description which picks out the Platonist's length-in-meters function (among all other functions from physical objects to real numbers) by specifying that it assigns length 1 to some canonical path and respects the following pair of nominalistic relations:

- $\leq_L$ 'path p_1 is at least as long as path p_2'
- $\oplus_L$ 'the combined lengths of path p_1 and p_2 together are equal to the length of path p_3'[12].

Thus, we have a formula ψ which picks out the Platonist's length-in-meters function at all words where length is richly instantiated. So at all such possible worlds a Platonist sentence $\phi(l)$ (in the language of set theory with ur-elements with l being a name for this length function) will be true if and only iff the corresponding nominalist sentence $P^*(\phi)$ (below) is true.

> $P^*(\phi)$ 'Necessarily if there are objects satisfying our description of the hierarchy of sets with ur-elements $V_{\omega+\omega}$ then $(\exists f)(\psi(f) \wedge \phi[l/f])$'

[11] Specifically, we can prove the uniqueness claim above holds whenever the following three principles (which all happen to be statable in the language of set theory with ur-elements) are satisfied.

Closure Under Multiples: Given a path x, there are paths y with lengths equal to any finite multiple of the length of x.

Archimedian Assumption: No path is infinite in length with respect to another, i.e., if $x \leq_L y$ then some finite multiple of x is longer than y (i.e. there's a path shorter than y, which can be cut up into n segments each of which has the same length as x.

Relational Properties: The relations $\leq_L, \oplus_L$ have the basic properties you would expect from their role as length comparisons.

My presentation follows [22].

[12] I will say a function $l(x)$ respects $\leq_L, \oplus_L$ just if for all paths a, b and c $a \leq_L b \iff l(a) \leq l(b)$ and $\oplus_L(a,b,c) \iff l(a) + l(b) = l(c)$.

Thus one might hope Platonist appeals to length relations can be harmlessly replaced by the strategy above. And maybe (as Field perhaps suggests in [9]) Platonist talk of mass, charge etc. functions could be handled similarly.

However, a difficulty which I'll call the Sparse Magnitude problem (and attempt to solve in this paper) arises. For, although lengths are plausibly richly instantiated in our world, it's not clear that they're richly instantiated at all metaphysically possible worlds. And other physical magnitudes, like mass and charge, don't even seem to be richly instantiated in the actual world. Indeed, as Eddon puts it [8]:

> It seems possible for there to be a world, w_1, in which a and b are the only massive objects, and a is twice as massive as b. It also seems possible for there to be a world, w_2, in which a and b are the only massive objects, and a is three times as massive as b. Worlds w_1 and w_2 are exactly alike with respect to their patterns of [how the relations 'less massive than' $o_1 \leq_M o_2$ and $\oplus_M(o_1, o_2, o_3)$ 'combined mass of a + mass of b = mass of c' apply]. And thus they are exactly alike with respect to the constraints these relations place on numerical assignments of mass. ... So it seems we cannot discriminate between the two possibilities we started out with.

These considerations threaten to block the above nominalist paraphrase strategy by showing that length is a special case. They suggest that other physical magnitudes (like mass) can't be pinned down in the same way that length can, and perhaps that the values of physical magnitudes don't supervene on facts about how *any* finite list nominalistic relations) apply[13]. Field notes and discusses a version of this problem in [10] the last chapter of[11].

[13]Thus version of Putnam's famous counting argument in [19] threatens to re-arise, even for those nominalists like Field in [9] who avoid the specific concern about lengths he mentions by accepting the existence of spatial points or paths.

4.2 Four Place Relation

I'll now argue that we can solve the above sparse magnitudes problem by using two cheap tricks. Specifically, suppose the Platonist worries that object masses or some other physical magnitude (given by real numbers) can't be captured by any relations between nominalisticly acceptable objects.

First, I claim that if we (temporarily) assume that length is richly instantiated at all possible worlds, we can solve the sparse magnitude problem by using the relationship between length and mass to pin down a mass assignment property (and likewise for other physical magnitudes).

For example, the nominalist can pick out a correct mass function by appeal to a four-place relation between pairs of objects with masses and pairs of paths:

- $M(o_1, o_2, p_1, p_2)$ which holds iff the ratio of the mass of o_1 to the mass of o_2 is $\geq$ the ratio of the length of path p_1 to the length of the path p_2.

Although such a relation may not be very physically (or metaphysically) natural, it reflects a genuine nominalisticly acceptable fact about the world. By the measurement theory results mentioned above, we can uniquely pin down the length function (up to a choice of unit), at all worlds where length is richly instantiated. Furthermore the claim that length is richly instantiated implies that, for any pair of distinct real numbers r and r', there is a pair of paths p_1 to p_2 whose lengths stand in a ratio that's in the interval between r and r'.

Thus, we can pick out the intended mass in grams function $\mathscr{M}$ (within our simulated hierarchy of sets with ur-elements) by saying that it assigns mass 1 to a suitable unit object and assigns mass ratios which bear the right relationship to the length ratios assigned by a correct length function. Specifically, we demand that any mass function $\mathscr{M}$ satisfy the constraint that if $\mathscr{L}$ is a length function respecting $\leq_L, \oplus_L$ then $M(o_1, o_2, p_1, p_2)$ holds iff $\mathscr{M}(o_1)/\mathscr{M}(o_2) \geq$

$\mathscr{L}(p_1)/\mathscr{L}(p_2)$[14]. This condition ensures that $\mathscr{M}$ assigns mass ratios correctly, provided length is richly instantiated.

This, in turn, is enough to allow us to apply the paraphrase strategy discussed above to claims involving a mass function (and the same goes for other physical quantities).

4.3 Holism trick

Now what about the above assumption that length is *metaphysically necessarily* richly instantiated? This assumption seems unmotivated but, happily, we can eliminate it if (as currently appears to be the case) our best scientific theory implies that length is *actually* richly instantiated[15].

To see how, consider some such platonistically formulated theory T which implies that space is richly instantiated.

By the considerations above we can produce a partially accurate paraphrase $P^*(T)$ which gets the correct truth-value at worlds where length is richly instantiated, but may get the wrong truth value at other possible worlds.

We can also write a completely correct nominalistic paraphrase of the claim that space is richly instantiated (call this R)[16].

Thus we can create a paraphrase which gets correct truth conditions for our theory at all possible worlds by simply writing the following conjunction.

P(T): $P^*(T) \wedge R$

At worlds where length is richly instantiated, $P^*(T)$ has the correct truth

[14] Consider any $\mathscr{M}'$ that attempts assign the wrong mass ratio r' to a pair of objects o_1, o_2 with mass ratio r. Any such function will fail to honor the true $\mathscr{M}(o_1, o_2, p_1, p_2)$ fact relating the ratio between the masses of o_1, o_2 to the ratio of length between a pair of paths p_1, p_2 such that $\mathscr{L}(p_1)/\mathscr{L}(p_2)$ falls between r and r'. And the existence of such a pair of paths is guaranteed by the assumption that length is richly instantiated, as noted above.

[15] A similar technique can plausibly be used to paraphrase physical theories that say that space is quantized because they tend to say that other physical magnitudes are also quantized. Thanks to REDACTED for this point.

[16] Note that this claim is statable using only set theory with ur-elements and the relations $\leq_L, \oplus_L$, so our basic modal if-thenist strategy suffices to paraphrase it.

value by our initial point, and R is true at those worlds, so the above conjunction will have the correct truth value. And at worlds where space isn't richly instantiated R is false, hence so is our paraphrase. Thus, in both cases, our paraphrase has the intended truth value.

Hence, the nominalist plausibly *can* address the sparse magnitude problems sufficiently well to answer the classic Quinean indispensability argument.

5 Conclusion

In this paper I have argued that we can plausibly answer classic Quinean indispensability worries about laws involving physical magnitudes as traditionally stated. I noted that by adding few formal cheap tricks to the modal if-thenist strategy for paraphrasing platonist theories, we can (plausibly) normalize scientific theories involving about physical magnitudes sufficiently well to answer the classic Quinean indispensability problem. I've also argued that (despite first appearances) these paraphrases have significant explanatory virtues and can plausibly be used to answer Baker's Explanatory indispensability worries as well.

However, this doesn't mean that things are smooth sailing for the nominalist. Even if we accept my cheap tricks for solving classic and explanatory indispensability arguments, physical magnitude statements do arguably still pose a reference and grounding indispensability problem for the nominalist. Although not needed to state our best scientific theories, mathematical objects may be indispensable to accommodate certain philosophical intuitions about reference and grounding. If there are no numbers, how are humans able to finitely learn languages which draw certain distinctions (which we intuitively can draw) between metaphysically possible worlds quite different from our own? And what could ground the truth of fundamental physical magnitude facts in the worlds?

If one accepts this revision of indispensability worries, some interesting consequences (for readers of different philosophical stripes) follow.

First, hardcore naturalists may be inclined stop taking indispensability worries (based on concerns about physical magnitude statements) seriously. For we see that the nominalist's real problem doesn't concern stating or (in a sense) attractively explaining *scientific* facts involving mass and charge, but rather accounting for certain a priori philosophical intuitions about metaphysical possibility, reference and grounding. Philosophical explanation is the sticking point, not scientific explanation.

Second, accepting the existence of mathematical objects on the basis of these Grounding and Reference Indispensability worries (if it turns out they cannot be answered) rather than on the basis of Quinean or Explanatory Indispensability arguments would have a few interesting consequences.

For one thing, if one accepts the existence of mathematical objects because of the above grounding challenge, then one has an automatic answer to certain access worries. I have in mind the suggestion [14] that if there hadn't been mathematical objects everything would have been the same. For, if mass facts are grounded in (and thus, plausibly, something like partly constituted by) a certain relation holding between physical objects and numbers, then the following (opposite) counterfactual intuition seems plausible: if numbers suddenly stopped existing then objects wouldn't have had masses, just as if hair suddenly stopped existing then people would stop having beards.

For another thing, consider arguments that we're only justified in believing mathematical objects exist contingently because our only reason for accepting their existence in the first place is the role they play in our best scientific theories (as per the Quinean Indispensability argument) [6]. The reference and grounding indispensability arguments mentioned above present a twist on the classic Quinean Indispensability argument which (if compelling) does justify the necessary existence of mathematical objects. In order to resolve the grounding and reference problems raised above, mathematical objects would need to exist necessarily.[17]

A Translation Strategy Details

To make my proposed basic modal if-thenist paraphrase strategy T more precise, I'll start by specifying some definitions used to state the definable supervenience condition described above.

A.1 Nominalistic vs. Platonistic Vocabulary

A relation R counts as **nominalistic** vocabulary iff the Platonist and nominalist agree that it only applies to non-mathematical objects. So, for example, 'is a cat' and 'is taller than' are nominalistic relations. Platonistic vocabulary is all vocabulary that isn't nominalistic. So for example 'is a number', 'is an element of', 'is a set of goats', 'is a function from the cats to numbers' and '...has more than...fleas' are all Platonistic vocabulary.

A.2 Categoricity Over

Next, we want to express the idea that some description D 'specifies, for each possible world w, exactly what mathematical objects the Platonist thinks exist at w (and how all relevant Platonistic vocabulary applies)', so that D can be a

[17] According to the grounding indispensability argument, grounding facts about mass ratios in a three place relation between a pair of objects and number best explains how objects stand in determinate mass ratios at remote worlds where neither length nor mass is richly instantiated. According to the reference argument, interpreting our talk of mass ratios in terms of a relationship between objects and numbers explains how our claims that objects have a certain determinate mass ratio can be true at such remote possible worlds. In each case, the explanatory role which mathematical objects are invoked to play directly requires that they exist necessarily (even in remote possible worlds).

suitable antecedent for our if then-ist translation.

First I will expand the notion of categoricity (all models of some theory are isomorphic) to a notion of **categoricity for** some list of relations **over** some other list of relations. I will say that a description $D(N_1, \ldots, N_m, P_1, \ldots, P_n)$ is categorical for the relations $P_1, \ldots, P_n$ over the relations $N_1, \ldots, N_m$ when (for every logically possible way the relations $N_1, \ldots, N_m$ could apply), requiring that D suffices to pin down a unique overall structure of objects satisfying relations in $P_1, \ldots, P_n, N_1, \ldots, N_m$. So stipulating that D uniquely determines (given the facts about how some relations nominalistic relations $N_1, \ldots, N_m$), how the objects related by these relations could be supplemented by additional objects satisfying platonistic relations $P_1, \ldots, P_n$[18].

For example, the following sentence D: SETS OF GOATS categorically describes how the Platonistic relations 'is a set-of-goats' and '...is an element of set-of-goats...' apply over the nominalistic relations 'is a goat'.

D: SETS OF GOATS

- The sets of goats are extensional[19].
- It's logically necessary, given the facts about how 'is a goat' 'is a set-of-goats' and '...is an element of set-of-goats...' are supposed to apply at any possible world, that if some goats are happy then there's a set of goats whose elements are exactly the happy goats.
- No goat is a set-of-goats.
- If x is an element of set-of-goats y, then x is a goat and y is a

[18] So, for example, if the sets of people, along with set membership, $(S_{\text{people}}, \in_{\text{people}})$ is categorical over the people P it's not just true that the number of sets of people is totally determined by what people exist but also facts such as whether or not any set of people is a person must also be determined. This claim can be nicely articulated in the language of logical possibility, as shown in REDACTED.

[19] That is, sets of goats a and b are identical just if they have exactly the same members.

set-of-goats.

]

A.3 Definable Supervenience

Now we can state the definable supervenience condition as follows.

A list of relations $\mathcal{P}$ **definably supervenes** (via a sentence D) on a finite list of nominalistic relations $\mathcal{N}$ iff

- There's a sentence D (a 'Supervenience Description' that intuitively explains how the relevant Platonistic facts supervene on nominalistic facts) in the language of logical possibility[20] which satisfies the following conditions
  - D is formed using only relations in $\mathcal{P}, \mathcal{N}$ and all quantifiers in D are restricted to objects that satisfy at least one relation in this collection[21]
  - The Platonist being translated takes D to express a metaphysically necessary truth.
  - $\Box\Diamond_{\mathcal{N}} D$, i.e., the Platonist isn't supposing the existence of incoherent objects and indeed it's logically necessary that the $\mathcal{N}$ structure can be supplemented with Platonistic structure in the way that D requires. $\mathcal{P}, \mathcal{N}$
  - D is is categorical for the relations $P_1, \ldots, P_n$ over the relations $N_1, \ldots, N_m$

For example, in the case above, note that the Platonist takes $D_{\text{Sets of Goats}}$ to be a metaphysically necessary truth. And $D_{SetsofGoats}$ specifies exactly what

[20]So D employs only the FOL logical connectives and the conditional logical possibility operator as logical vocabulary, and does not quantify in to the $\Diamond$ of logical possibility[3]

[21]The latter assumption ensures that D 'only talks about' the structure of objects satisfying relations in P and N.

sets of goats there are at each metaphysically possible world w (and how the elementhood relation these sets of goats), given the facts about what goats there are at each world. Also, it's logically necessary that, however the goats are configured, they can be supplemented with sets as required by $D_{\text{Sets of Goats}}$.

Surprisingly many collections of Platonistic sentences involving pure mathematical structures (of reals, complex numbers etc.) and applied mathematical objects (of classes of physical objects, functions from physical objects to pure mathematical objects) straightforwardly satisfy this definable supervenience condition.

[For another example, we can create a definable supervenience description D for translating sentences that talk about both numbers and the objects satisfying some nominalistic relations $\mathscr{N}$, by conjoining claims that the natural numbers are distinct from all the objects related by these nominalistic relations with a sentence $PA_{\Diamond}$ that categorically describes the natural numbers over N.

And we can write a sentence $PA_{\Diamond}$ (using the conditional logical possibility operator) that categorically describes the intended structure of the natural numbers $\mathbb{N}, \mathbb{S}$ over any list of relations, including the empty list of relations[22] as follows. First recall] that can categorically describe the natural numbers via the second order Peano Axioms, a combination of all the first order Peano Axioms except for instances of the induction schema conjoined with the following second order statement of induction.

$$(\forall X)\,[(X(0) \wedge (\forall n)\,(X(n) \rightarrow X(n+1))) \rightarrow (\forall n)(X(n))]$$

We can reformulate this claim using conditional logical possibility as follows[23].

[22] Unsurprisingly we don't need to appeal to facts about how any nominalistic relations happen to apply in order to pin down the intended structure of these pure mathematical objects.

[23] I write '0' below for readability, but recall that one can contextually define away all uses

- '$\Box_{\mathbb{N},S}$ If 0 is happy and the successor of every happy number is happy then every number is happy.

In other words: it is logically necessary, given how $\mathbb{N}$ and S apply, then if 0 is happy and the successor of every happy number is happy then every number is happy.'

Thus, we can write a sentence $PA_{\Diamond}$, (purely in terms of first order logic plus the conditional logical possibility operator) which categorically describes the natural numbers. Just use the fact above to replace the second-order induction axiom in second order Peano Arithmetic with a version stated in terms of conditional logical possibility. Recall that the Second Order Peano Axioms are the familiar first order Peano Axioms for number theory, but with the induction schema replaced by a single induction axiom using second order quantification. In [3] Berry argues that we can similarly rewrite other second-order conceptions of pure mathematical structures.]

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