# PHYSICAL POSSIBILITY AND DETERMINATE NUMBER THEORY

SHARON BERRY

Abstract. It's currently fashionable to take Putnamian model theoretic worries seriously for mathematics, but not for discussions of ordinary physical objects and the sciences. But I will argue that (under certain mild assumptions about the physical possibility of infinite stochastic physical systems) merely securing determinate reference to *physical* possibility suffices to rule out nonstandard models of our talk about number theory. So anyone who accepts realist reference to physical possibility faces pressure to also accept such reference to (at least) the standard model of the natural numbers.

## 1. Introduction

Putnam famously used the possibility of nonstandard models to raise a challenge for realist reference to mind-independent objects, and realist claims to have a categorical conception of the structure of the natural numbers. It's currently fashionable to take such model theoretic worries seriously for mathematics, but not for discussions of ordinary physical objects and the sciences [10, 2].

I will attack this combination of views by argueing that (under certain mild assumptions about the physical possibility of infinite random physical systems) merely securing determinate reference to *physical* possibility suffices to rule out nonstandard models of our talk about number theory. So anyone who thinks we can determinately refer to physical possibility faces pressure to also accept such reference to (at least) the standard model of the natural numbers.

## 2. Putnam's Model-Theoretic Challenge

Let me begin by laying out Putnam's model-theoretic challenge, as it applies to the natural numbers.

The standard first order axioms of arithmetic (PA) plausibly articulate part of our concept of numbers (by constraining how the relations $\mathbb{N}, S, +, *, <$ apply). However, these axioms can also be satisfied by non-standard models, which have a different structure and may change the truth-value of some arithmetic sentences. For instance, PA requires that every number besides 0 both have and be a successor. However, this principle (on it's own) leaves open the possibility of a non-standard model which (under $<$) looks like the following (where each additional $*$ indicates a disjoint copy of the integers):

$$0, 1, 2, 3, \ldots, -2^*, -1^*, 0^*, 1^*, 2^*, 3^*, \ldots, -2^{**}, -1^{**}, 0^{**}, 1^{**}, 2^{**}, 3^{**}, \ldots$$

The resulting structure looks like a copy of the natural numbers followed by two copies of the integers. Note that, by ensuring there is no least 'infinite' number, such a structure can satisfy the requirement that every 'number' besides 0 both is and has a successor.

This alone isn't enough to create a non-standard model of $PA$[1]. However, if we instead consider the structure consisting of a copy of the natural numbers followed by infinitely many copies of the integers *densely ordered* (i.e., the resulting structure has the form $\mathbb{N} + (\mathbb{Z}) \cdot Q$ where $\mathbb{Z}$ is just the integers and $Q$ is the rationals)[7], then there is a way for the relations $+, *, <$ to apply so that all so that all the Peano axioms are satisfied

[1]Note that we've only said that the basic first order Peano axioms about how $S$ applies are satisfied in the structure above, not that any of the other ones (i.e., those for $+, \cdot$ and $<$) are.

– including all instances of the first order induction schema[2]. Such non-standard models of PA don't satisfy the full second order induction axiom, $(\forall X)\,[(X(0) \wedge (\forall n)(X(n) \rightarrow X(S(n)))) \rightarrow (\forall m)X(m)]$[3]. So they don't satisfy second order Peano Arithmetic[4]. But they do satisfy all the axioms in PA, i.e., those of first order Peano Arithmetic (and this implies that they have a standard initial segment, something which will be important later on).

In view of the existence of such non-standard models, one can ask (as Putnam does) the following question. Do we really have a definite concept of 'the structure of the natural numbers' which is not satisfied by any non-standard models? What can such a concept consist of? What is it about us which (perhaps together with other kinds of facts about the world) lets us our words like "number" and "plus" take on meanings which rule out such non-standard models? For reasons I won't discuss here, Putnam takes our ability to give standard meanings to the first order logical vocabulary for granted in his challenge. I will follow him in doing so.

Accordingly, we can dramatize Putnam's challenge as follows. Imagine some all-knowing interpreter who is dedicated to interpreting our talk about the natural numbers in some unintended fashion while preserving the meaning of all first order logical vocabulary. This mischievous interpreter has full access to ordinary determinate mathematics and uses that knowledge to construct non-standard models for our talk of the natural numbers to refer to.

---

[2]That is, all sentences of the following form in the language of arithmetic $\phi(0) \wedge (\forall n)[\phi(n) \rightarrow \phi(S(n))] \rightarrow (\forall m)\phi(m)$.

[3]Indeed, the set $X$ consisting of just the standard integers (i.e., the objects represented by '$1, 2, 3 \cdots$' in the example above) satisfies $(\forall n)[X(n) \rightarrow X(S(n))]$ but doesn't satisfy $(\forall n)X(n)$

[4]By this I mean the theory which replaces all instances of the induction schema in familiar first order Peano Arithmetic with the the second order induction axiom above.

Can we cite plausible constraints which our mischievous interpreter must honor which prevent him from giving an unintended interpretation? Note that classic results in mathematical logic [6] tell us that merely requiring he make some algorithmically listable set of axioms come out true (e.g., axioms extending PA or embedding the numbers in a larger structure) couldn't provide such a constraint.

If we can give no satisfying answer to Putnam's challenge, then, perhaps, we must allow that our conception of the structure of the natural numbers is vague and allows for a range of acceptable precisifications (corresponding to different structures satisfying the Peano Axioms, much like the range of acceptable precisifications of 'bald' and 'heap')[5]. But admitting this raises problems for the common presumption that all statements of arithmetic (even ones we can't decide) have definite truth-values. For the most common way of ensuring such definite truth values is through reference (up to isomorphism) to the natural numbers[6]

Now Putnam supplements this mathematics-specific worry with a more general model theoretic challenge to realist approaches to truth and reference. Very crudely, the worry goes like this. From a naive realist point of view, it seems that we can talk about physical objects and grasp scientific concepts in a way that makes it possible for a an ideal scientific theory[7] to be wrong. However, any consistent first order theory can be interpreted as speaking truly about (some of) the sets. So why doesn't this theory count

[5]Though, as Hartry Field[4] has emphasized, taking such a position would still allow us to use classical logic when reasoning about the natural numbers (because, e.g., formulas of the form '$P \vee \neg P$' will be true on all acceptable precisifications).

[6]More directly, Putnam's model theoretic challenge raises problems for truthvalue realism about mathematics as follows. Combining the completeness and incompleteness theorem tells us that any first order algorithmically listable theory of the numbers (extending PA) will have models which disagree on the truth of some number theoretic claim. So if we concede to the Putnamian skeptic that any model of (some extension of) PA is an equally acceptable precisifications of our number concept then we are forced to conclude that the truth-value of some number theoretic claims is vague or indeterminate.

[7]That is a theory that would be accepted in the 'ideal limit' of scientific investigation.

as speaking truely of this model? Why isn't it more charitable to interpret this theory as speaking truly of the sets rather than falsely of electrons and rabbits (the apparent subject matter of the theory)?

As noted above, many philosophers are inclined to take model theoretic worries about grasping mathematical concepts seriously, while rejecting this more general challenge. Perhaps this difference can be partly motivated by the fact one can invoke causal contact with objects like rabbits and electrons in answering Putnam's general challenge. But since mathematical objects are generally taken to be causally inert we cannot do the same when answering Putnam's challenge with respect to the natural numbers.

I will now argue against this combination of views by showing that merely taking us to (somehow) have a determinate grip on the concept of physical (or metaphysical possibility) provides sufficient resources to answer Putnam's challenge concerning arithmetic. Our ability to form a definite notion physical (or metaphysical) possibility provides a kind of failsafe which is sufficient to rule out non-standard interpretation of our talk about the numbers on its own.

## 3. Contrast With Open-Endedness and Temporal Approaches

Let me begin by quickly reviewing the two closest proposals to mine in the existing literature.

3.1. **The Language Expansion Approach.** In [9] [8] Parsons and McGee have offered an answer to the Putnamian challenge centering on what McGee calls openendedness. Openendedness is the idea that we expect all instances of the first order induction axiom schema to continue holding true in any 'logic preserving' extension of our language.

McGee argues (roughly[8]) as follows. Part of our current use of number talk is to expect that the induction schema will remain true in all 'logic preserving' expansions of our language. McGee suggests that this fact helps rule out non-standard models as follows. Suppose (for contradiction) that some nonstandard model $M$ provided an acceptable interpretation of our terms 'natural number', 'successor' etc. Then there could (in some sense) be a god who is able to point to the non-standard model and introduce a term "smee" which applied counter-inductively to this non-standard model (i.e., smee applies to 0, and $smee(n) \rightarrow smee(S(n))$, but smee doesn't apply to every 'natural number'). If we met such a god then we could (logic-preservingly) extend our language by taking the term 'smee' from their language and adding it to ours. In such a case, we would still expect the induction axiom to hold for formulas involving smee which we got from talking to this god. Therefore, interpreting us to mean a nonstandard model is unacceptable because it would fail to satisfy induction in this extended language.

This strategy faces a number of objections. First, it might well be metaphysically impossible for a god to introduce a term like smee. For instance, it's not clear how the god could refer sufficiently definitely to some proper initial segment of our non-standard model. What can the god do to secure reference in a way we cannot? Are we to imagine a metaphysically impossible scenario where they fly into the realm of abstract objects and point one by one to each of the infinitely many elements in the initial segment? Maybe we should imagine they perform some supertask with physical objects that pins down this initial segment[9]. Perhaps this objection is close to what Field

[8]McGee's actual proposal is somewhat more complicated in ways that I claim don't effect any of the criticisms discussed here. See pgs 56-68 of [8] for the details I've elided.

[9]If the god just introduces standard explicit definitions, this doesn't seem to increase the expressive power. Maybe we are supposed to imagine them introducing a truth predicate for our language and then another on and on. But there's much debate over what happens

had in mind when he expressed a worry like, 'why can't we just say that we secure definite reference by whatever we are imagining the god to do to secure her reference?' in [4] [10] .

I don't think McGee would be much troubled by this worry, because he seems happy to accept the metaphysical impossibility of the scenarios he envisages and instead appeals to the idea that we are committed to the first order induction schema being true in all *logically* possible extensions of our language. He writes:

> To say what individuals and classes of individuals the rules of our language permit us to name is easy: we are permitted to name anything at all. For any collection of individuals K there is a logically possible world-though perhaps not a theologically possible world-in which our practices in using English are just what they are in the actual world and in which K is the extension of the open sentence 'x is blessed by God'. So the rules of our language permit the language to contain an open sentence whose extension is K[8].

---

with such truth predicates. Relatedly, the usual answer to Kaplan's paradox (if there are $\alpha$ worlds then there are $2^\alpha$ possible propositions so, e.g., it can't be the case that for each proposition there is a distinct possible world at which only that proposition is expressed) provides strong reason to think many 'combinatorially possible' ways a language could work are actually *not* metaphysically possible for anyone to have.

[10]Field writes, "...how can adopting McGee's rich view of schemas help secure determinacy? That view of schemas merely allows me to add an instance of induction whenever I add new vocabulary. But the relevant vocabulary for McGee's argument would seem to be 'standard natural number', and we've already seen that that is no help. Of course, it's true that if I could add a predicate that by some magic has as its determinate extension the genuine natural numbers, then I will be in a position to have determinately singled out the genuine natural numbers. That's a tautology, and has nothing to do with whether I extend the induction schema to this magical predicate. But if you think that we might someday have such magic at our disposal, you might as well think we have the magic at our disposal now; and again, it won't depend on schematic induction. So the only possible relevance of schematic induction is to allow you to carry postulated future magic over to the present; and future magic is no less mysterious than present magic."

However, one might worry that our dispositions in metaphysically impossible scenarios like the above are not clearly enough understood to be invoked in this context. Thus one might worry that such counter-possible conditionals don't so much explain our ability to determinately refer to the natural numbers as package the intuition that we do.

One might also object that availing ourselves of the space of all logically possible extensions of our language to explain how we have a determinate conception of the natural numbers is question begging. We wouldn't accept an answer to Putnam's worries that *just presumed* we have a determinate conception of second order quantification and it's not clear that considering all logically possible linguistic extensions is materially different. If we can somehow intend that the induction schema remain true in all logic preserving expansions of our language (in the above sense which includes languages corresponding to all possible ways of choosing a subset of individuals for a predicate to apply to), why can't we use the same faculty to directly intend that our second order quantifiers range over every possible subset? One might also doubt that we even have a definite conception of logic preserving extensions of our language.

3.2. **Appeal to the Structure of Time.** In [3] Hartry Field (rather ambivalently) proposes an alternative account, on which he argues that *if* time (starting from any point) forms a genuine $\omega$ sequence[11] (i.e., time has infinite duration and there are only a finite number of seconds between any two times) then this belief can be used to rule out nonstandard interpretations of our number talk (given standard interpretations of our temporal

[11] An $\omega$ sequence refers to a collection of elements which, under some relation $<$, has the same structure as the intended model of the natural numbers, i.e., is comprised of a first element, the successor of that element and so forth. Note that the claim time forms an $\omega$ sequence (assuming it is linearly ordered) is equivalent to the claim that if we start marking off one second intervals at any point those marks form an $\omega$ sequence.

and event talks). I don't think this proposal works, even if the structure of time in our universe does happen to form a genuine $\omega$ sequence in the way Field imagines. For we treat the assumption that time forms a genuine $\omega$ sequence as (at best) a contingent hypothesis, and not a conceptual truth constraining what we mean by the natural numbers. So it's not clear that any acceptable interpretation of our language must make the structure of the numbers come out the same as the structure of time. If I believed that the number of gumballs in the jar is 70, this belief presumably wouldn't commit a mischievous interpreter to interpret the term 'natural number' to make this statement true. So why would the above conjecture (about the relationship between the natural number structure and that of time) have any more power to constrain acceptable interpretations of our natural number concept? One might also worry about Field's talk of isomorphism and functions given that we cannot take the meaning of second order quantifiers for granted[12].

## 4. My Proposal

4.1. **Expectations About Possibility.** I will now present a different kind of answer to Puntam's model-theoretic challenge for the natural numbers. Specifically, I will argue that if we are are somehow able to latch on to a notion of *physical possibility*, and certain kinds of infinite sequences of random events (to be specified below) are physically possible, this suffices to rule out nonstandard models of number theory. Thus, I will argue that grasping a notion of physical (or metaphysical possibility) provides a kind of failsafe or back door to grasping this notion.

[12]However if Field accepted something like the trick I propose below, then we could go from the fact that time is actually an $\omega$ sequence realist reference to time and expectations that no definable predicate applies counter-inductively to an ability to mean the standard models.

4.2. **Assumptions.** Let me begin by laying out the following key assumptions which my argument requires. First, we need a bunch of fairly bland assumptions to cash out the idea that we are taking reference to physical necessity for granted and imposing some basic constraints on our interpreter.

I will say that an interpreter is **well behaved** iff they satisfy the following criteria:

- They select a single model as the referent of our concept 'natural number' in all physically possible worlds.
- They cannot tamper with extension of the following non-mathematical vocabulary: 'coinflip' 'heads' 'temporally after' at any of these possible worlds.
- They give the usual meaning to logical vocabulary and the physical necessity operator, e.g., the existential quantifier and the physical necessity operator $\Box_p$ must contribute to truth conditions in the usual fashion.
- They must make all statements which we are willing to endorse as conceptually required by our grasp of the natural numbers (such as the Peano axioms) come out true. Note that this requires that the induction holds for formulas defined using non-mathematical as well as mathematical properties[13]. Thus, if $Q(n)$ abbreviates "there is an n-th coinflip, and the n-th coinflip comes up heads,", then from $Q(0) \wedge (\forall n)\,[Q(n) \implies Q(S(n))]$ we can infer $(\forall n)Q(n)$
- They vindicate all sentences we treat as conceptual truths relating the numbers and the practice of counting a sequence of events in time (specifically it suffices to vindicate those truths specified in section 4.3).

[13]More restrictively, it suffices to augment the mathematical vocabulary with the terms in 4.2 and 4.2

Now I will argue that, if it is physically possible for there to be an infinite series of random events then any well-behaved interpreter must interpret us as talking about a standard model of the natural numbers. I don't claim it is obviously true that such a sequence is possible. However, I think it is plausible enough to pose a substantial problem for those who combine realism about physical possibility with indeterminacy about our conception of the natural numbers.

> Infinite Random Sequence (IRS): It is physically possible to have a series of independent *objectively* random events linearly ordered in time[14] with two possible outcomes having a first event but no final event. Furthermore, every event in the series has a temporal successor, i.e., for any event $x$ there is some other event $y$ occurring after $x$ such that no event $z$ occurs between $x$ and $y$.

Informally, one can think of the events whose possibility IRS asserts as being like the ticks of an indestructible watch which never needs repair or winding. There is a first tick, each tick is followed by a unique next tick and there is no tick after which the watch breaks down.

To motivate accepting this principle, note that it is only asserting that it is physically possible to repeatedly perform (independent) textbook spin measurements on an electron[15][1] (or some other equivalent process) and that the laws of physics don't rule out time continuing infinitely into the

[14]That is for any distinct events $x, y$ in the series either $x$ occurs before $y$ or $y$ occurs before $x$. Moreover, from the point of view of relativistic physics, the measurements are separated by time-like intervals ($x$ is in the future light cone of $y$ or vice versa) so all observers agree on their order. Given these constraints it is safe to simply work relative to some fixed inertial reference frame and ignore relativistic complications for the remainder of the paper.

[15]That is perform a spin measurement along the $x$-axis on an electron whose spin has just been measured (and thus collapsed) along the $y$-axis. Thanks to REDACTED for suggesting these details.

future (though possibly having non-standard 'length')[16]. I will abstract away from the details of the measurement and simply refer to it as a coinflip and the two outcomes as heads and tails.

4.3. **Banishing Non-standard Models.** With these assumptions in place, we can finally turn to foiling a mischievous (but well behaved) interpreter. My argument will turn on the following key claim, which I take to be as conceptually core[17] to our notion of the natural numbers as $PA$ is (and hence once of the conceptual truths which our well behaved interpreter must honor). That claim is that the induction axioms applies to the predicate the $n$-th coinflip landed heads.

To write this claim up carefully in first order logic, note that our current mathematical language allows us to use natural numbers to talk about events taking place in time such as 'the 4th U.S. President' or 'the 37th successful rickrolling'. This practice of talking about the $n$th coinflip presumably includes accepting principles like, 'if no coinflip occurred before $x$, then $x$ is the 0th coinflip.' I take the following such principles to be conceptual truths regarding counting (temporal) sequences of events using the natural numbers[18], where $coinflip(x)$ denotes $x$ is a coinflip, $countflip(n, x)$ denotes $x$ is the $n$-th coinflip, $heads(x)$ denotes that coinflip $x$ has the heads outcome and $before(x, y)$ denotes that the coinflip $x$ occurs temporally prior to coinflip $y$.

- An object $x$ is the 0th coinflip, i.e., $countflip(0, x)$ iff $x$ is a coinflip and all other coinflips happen after $x$. $(\forall x)[countflip(0, x) \leftrightarrow coinflip(x) \wedge (\forall y)(countflip(y) \rightarrow before(x, y) \vee x = y))]$

[16]We will see that, ultimately, the use of objective randomness is just a way to establish it would be physically possible for there to be a temporal $\omega$ sequence of of objects satisfying some property (having a determinate extension) in our current language.
[17]Note that being conceptually core could be a matter of degree [11].
[18]C.f. [5].

- If $x$ is the $n$th coinflip, then $y$ is the $S(n)th$ coinflip iff $y$ occurs after $x$ and no other coinflip occurs between $x$ and $y$. That is,

$$(\forall n)(\forall x)(\forall y)(countflip(n,x) \rightarrow$$

$$[(countflip(S(n),y) \leftrightarrow coinflip(y) \wedge before(x,y) \wedge (\forall z)\neg\,(coinflip(z) \wedge before(x,z) \wedge before(z,y))]$$

- Only coinflips can be the $n$th coinflip, i.e., $(\forall x)(\exists n)(countflip(n,x) \rightarrow coinflip(x))$
- No two distinct numbers correspond to the same coin flip. $(\forall n)(\forall m)[coinflip(n,x) \wedge coinflip(m,x) \rightarrow m = n]$

I take it that we accept all these principles, and take them to apply with physical (and metaphysical and logical etc) necessity. So we accept $\Box_P$(COUNTING RULES), where COUNTING RULES is the conjunction of the claims above. Furthermore, I take this modal claim ($\Box_P$(COUNTING RULES)) to be conceptually core in the sense mentioned above – so that our mischievous interpreter must make it come out true.

Now suppose the mischievous interpreter wants to take us to refer to some non-standard model of the numbers. Together with IRS, the above conceptual truths regarding counting ensure[19] that there is a physically possible world at which our current vocabulary picks out a counter-inductive[20] collection of numbers (thereby witnessing that the restrictions on our interpreter should have prevented that choice of non-standard model).

[19]Note that in many physically possible situations there will be a 'number' $n$ such that these analyticities plus the facts about how $countflip()$, $coinflip()$ and $before()$ apply insure that there is no $n$th coinflip for certain values of $n$. For example, if no coinflips take place after the nth coinflip there will be no $n+1$th coinflip. Even in worlds whose possibility is asserted by IRS it might be that there are only standard temporal durations, e.g., $n$-seconds after only makes sense for standard integers $n$, in which case those worlds wouldn't have any $n$-th coinflip where $n$ is non-standard.

[20]These conceptual truths do not necessarily uniquely determine which elements $n$ in the nonstandard model will be interpreted to satisfy '$(\exists x) coinflip(x,n) \wedge heads(x)$ but they insure that all such interpretations will be counter-inductive.

To see this, note that by IRS there is a physically possible world $w$ where infinitely many coinflips (linearly ordered by temporally before) take place and all and only the initial $\omega$ sequence of these coinflips come up heads. I claim that the above constraints on the mischievous interpreter ensure that she takes $P(n) \stackrel{\text{def}}{=} (\exists x)(countflip(x, n) \wedge heads(x))$ to hold for just those $n$ in the standard initial segment of the nonstandard referent of the natural numbers – so that induction fails at this physically possible world for the property $P(n)$.

The mischievous interpreter can't simultaneously satisfy the induction axiom for P, and the principles governing countflip above (while taking $coinflip()$ and $heads()$ $before()$ to have their intended extension) at this troublesome world! To see why, imagine her predicament when choosing an extension for 'countflip' in $w$. The principles governing countflip tell us that 0 has to be assigned to the temporally first coinflip in $w$, 1 to the next, and so on for all the objects in the standard initial segment of the nonstandard model. This uses up all the coinflips landing heads (which by our principles can't be reused) ensuring that whenever $P$ applies to $n$ it also applies to $P(n)$. Hence, $P$ must apply to all natural numbers contradicting the non-standardness of the interpretation.

So (to summarize) if IRS is true than (provided coinflip(), heads(), before() have their usual interpretation) either 'natural number' is interpreted standardly or an instance of the induction schema fails to hold with physical necessity. Note that this argument only requires we can understand (and accept) certain claims about physical necessity and does not question-beggingly presume that we can antecedently distinguish worlds at which only an initial $\omega$ sequence of coinflips come up heads.

Moreover, exactly the same argument given above would work if we replaced appeal to a definite notion physical possibility $\Box_p$ with appeal to a definite notion of metaphysical possibility $\Box_m$ – with the cheering improvement that the version of IRS which merely asserts metaphysical rather than physical possibility no longer depends on contingent physical assumptions to ensure its truth. After all, it is clearly metaphysically possible for there to be an infinite random sequence[21][22].

## 5. Conclusion

In this paper I have argued that the currently fashionable combination of taking Putnamian model theoretic worries seriously for mathematics, but not for discussions of ordinary physical objects and the sciences is unsatisfactory. Specifically I have argued that we can appeal to expected relationships between mathematical facts and physical (possibility) facts to rule out non-standard models of our number theoretic talk.

Let me close on a note of humility with two caveats. First, I admit that philosophers of mathematics committed to the idea that there are determinate right answers to all questions in number theory will likely also doubt

[21]Realtedly, one can note that my argument for number theoretic determinacy doesn't requires supposing that onto a completely determinate notion of physical possibility. Our concept of physical possibility might be vague or underspeficied in a number of different ways, allowing for many different acceptable sharpening. All my argument needs is the presumption all acceptable precisifications agree in taking the infinite sequence of random events invoked in the previous section to be physically possible.

[22]Also note that this proposal avoids the objection to Field's approach to securing number theoretic determinacy via physical determinacy (that ideal interpreters needn't and intuitively shouldn't constrain their understanding of number talk to make all our guesses about numbers of gumballs and the structure of time). Admittedly both stories require an assumption about contingent physical facts (about the structure of time, in his case, and about the possibility of infinite random sequences, in my case). But I am saying that if physical facts are a certain way (and standard interpretations for physical vocabulary are secured) then a deviant interpreter can't make certain sentences which we take to express conceptual truths (e.g. the claim that a certain instance of the induction schema holds with physical/metaphysical necessity) come out true. Field is saying that if physical facts are a certain way then a deviant interpreter can't make certain sentences which we take to be conceptually peripheral guesses (e.g., that there are only finitely many seconds between any two times) come out true. It is far clearer that an ideal interpreter should constrain their interpretation of 'natural number' to make the former come out true than the latter.

that we can have the determinate grip on physical and metaphysical possibility which my response assumes. My aim in this paper has only been to argue that a certain fashionable combination of views (determinacy about physical or metaphysical possibility combined with indeterminacy about mathematics) is unstable, not to show that one should resolve this tension by taking a realist position on both issues.

Second, although I have argued that leveraging physical possibility offers a route to grasping a definite concept of the natural numbers, I don't mean to suggest that this is the only or primary way that we can grasp such a concept. It would be strange if our possession of a definite conception of the natural numbers depended on our beliefs about physical (or metaphysical) possibility. Thus, I think that another - rather different- style of answer to Putnam's challenge must also be possible.

## References

[1] David Z. Albert. *Quantum mechanics and experience*. Harvard University Press, 2009.

[2] Tim Button and Sean Walsh. Structure and categoricity: Determinacy of reference and truth value in the philosophy of mathematics. *Philosophia Mathematica*, 24(3):283–307, 2016.

[3] Hartry Field. Which undecidable mathematical sentences have determinate truth values? *Truth in Mathematics*, pages 291–310, 1998.

[4] Hartry Field. *Truth and the Absence of Fact*. Oxford University Press, 2001.

[5] Gottlob Frege. *The Foundations of Arithmetic*. Evanston: Ill., Northwestern University Press, 1953.

[6] K. Gödel. Über formal unentscheidbare sätze der principia mathematica und verwandter systeme i. *Monatshefte für Mathematik*, 38(1):173–198, 1931.

[7] Richard Kaye. *Models of Peano Arithmetic*, volume 15 of *Oxford Logic Guides*. Oxford University Press, New York, 1991. Oxford Science Publications.

[8] Vann McGee. How we learn mathematical language,. *The Philosophical Review*, 106, 1997.

[9] Charles Parsons. The uniqueness of the natural numbers. *The Jerusalem Philosophical Quarterly*, 39:13–44, 1990.

[10] Jared Warren and Daniel Waxman. A metasemantic challenge for mathematical determinacy. *Synthese*, pages 1–19, forthcoming.

[11] Timothy Williamson. *The Philosophy of Philosophy (The Blackwell / Brown Lectures in Philosophy)*. Wiley-Blackwell, Oxford, 2008.