DEFAULT REASONABLENESS AND THE MATHOIDS

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1. Introduction

Where, if anywhere, do the chains of justification that support a priori knowledge come to an end? Philosophers who maintain that a priori knowledge is possible (and accept that there are definite context-independent facts about justification) have three options. We can allow that some foundational beliefs are justified without appeal to any other beliefs, we can allow that some circular arguments provide justification, or we can allow that some infinite descending chains of beliefs provide justification. If we take the first route there are again two possibilities: items of foundational a priori knowledge may be justified by appeal to something other than a belief, such as an inner experience of rational insight, or they may need no justification at all.

In the recent literature there has been much interest in the latter possibility: that certain 'default reasonable' (token) beliefs can be justified without appeal to anything at all [Wright, 2004, Field, 2000, Schecter and Enoch, 2008][1]. Invoking default reasonable beliefs as the source of a priori knowledge would allow us to avoid familiar objections to more traditional answers to the question about the foundations of a priori knowledge posed above. Both coherentist appeals to virtuous circles and rationalist appeals to inner experience

[1]Field defines "default reasonable propositions" as "propositions reasonably believed without any justification at all"[Field, 2000] and gives a similar account of default reasonable inference rules. I am simply extending this terminology to apply to particular token beliefs. This modification allows us to discuss the possibility that something other than the content of a token belief can be relevant to whether that belief counts as default reasonable. In this paper, I will largely talk about what makes beliefs default reasonable, however parallel arguments apply to default reasonable inferences and where necessary the term can be read to incorporate both beliefs and inference procedures.

have struck many philosophers as implausible or mysterious[2]. But, if all a priori knowledge can ultimately be grounded in default reasonable beliefs, then we can make sense of a priori knowledge without invoking either rationalism or coherentism.

In this paper, I will argue that (principled) attempts to ground a priori knowledge in default reasonable beliefs cannot capture certain common intuitions about what is required for a priori knowledge. I will describe hypothetical creatures who derive complex mathematical truths like Fermat's last theorem via short and intuitively unconvincing arguments. Many philosophers with foundationalist inclinations will feel that these creatures must lack knowledge because they are unable to justify their mathematical assumptions in terms of the kind of basic facts which can be known without further argument. Yet, I will argue that nothing in the current literature lets us draw a principled distinction between what these creatures are doing and paradigmatic cases of good a priori reasoning (assuming that the latter are to be grounded in default reasonable beliefs). I will consider, in turn, appeals to reliability, coherence, conceptual truth and indispensability and argue that none of these can do the job.

In making this case, my aim is not to attack default reasonableness approaches to the a priori, but rather to support the idea that there is no

[2]Here is a sample of traditional objections. If circular arguments in general cannot provide any justification, how can sufficiently large ones do this? Even if you allow that circular arguments and internal coherence can provide some degree of justification, one might think that we have more secure knowledge of the fact that there are infinitely many primes than we could have merely on the basis of such circular arguments. Also, one might think that people who are not disposed to give any justification for, say, the principle that everything is self identical still count as having knowledge of that claim. With regard to rationalism there are worries that no convincing non-mystical story about rational insight could be given. Questions can also be raised about whether rationalist appeals to inner experience really avoid any obstructions that arise for plain appeals to default reasonableness. It seems odd to claim that a brutely accurate mental pictures can provide more justification than brutely obvious non-pictorial thought, e.g., the claim that everything is self identical.

principled epistemically significant difference between ourselves and hypothetical creatures who accept intuitively inadequate arguments, by showing that at least one popular approach to the foundations of a priori knowledge makes it very difficult to locate any such difference. I will conclude by sketching some reasons why alternatives to the default reasonableness approach also appear to face trouble locating a principled sense in which we are better off than the creatures in my thought experiments, and exploring what the philosophical landscape looks like if we are not better off than them.

2. A Thought Experiment

A wide range of different propositions can feel immediately obvious. Think of what it feels like when you add $2 + 3$ and get 6, or how past generations felt when they assumed that (distinct) lines in space necessarily intersect at most once. In view of this possibility, I want to propose a thought experiment involving creatures who find, not falsehoods, but additional truths immediately obvious.

Imagine that we modify certain people, who we will call the the mathoids, to find a useful and mutually illuminating selection of theorems from the literature immediately obvious in phenomenologically much the same way the way that we find $0+1 = 1$[3] or the principle of mathematical induction immediately compelling. So, for example, they find Fermat's last theorem (FLT) immediately obvious and compelling. And since the mathoids find FLT immediately obvious they find it unnecessary to search for the kind of proof that Andrew Wiles produced [Wiles, 1995]. Suppose that the mathoids are modified in first grade, and then sent off to a colony with other mathoids and never told about their modification. Thus, we don't give them evidence of

[3]If you don't find this claim immediately compelling, consider instead the premise from which you derive $0 + 1 = 1$.

the reliable process by which we modified their mathematical intuitions, but they also don't get special reason to doubt their intuitions from interactions with peers who lack these intuitions.

Now that we've met the mathoids we can ask: do they count as knowing the claims which they find immediately obvious?

A number of people who are attracted to foundationalism about a priori knowledge have the intuition that the mathoids must lack knowledge, despite the truth of their beliefs and the reliability of their inferences. To motivate this idea, note that the mathoids hold their beliefs without being able to provide any argument which we would find convincing or point to any special faculty which they have and we lack. Accordingly, when the mathoids believe a proposition on the basis of deriving it from Fermat's last theorem it can be natural to think that they don't qualify as having knowledge of it (as opposed to the conditional asserting the proposition is true if FLT holds) unless and until they are able to 'fix the hole' in their proof by providing an argument justifying FLT. Supporting the intuition that the mathoids don't know is the use (without comment) in the literature of similar situations as a reason to reject theories of a priori knowledge. For example, as we will see below, Boghossian [Boghossian, 2003] rejects a rival theory on the grounds that it effectively[4] allows us to assume a complex, highly non-obvious mathematical truth.

Philosophers who have the intuition that the mathoids don't know will argue that there is a principled and epistemically significant difference between the sense in which trained mathematicians can gain a knowledge from arguments which 'skip steps' and what the mathoids do. When questioned,

[4]More precisely the theory allows us to gain knowledge by deploying necessarily truth preserving inferences (without further justification) which allow us to easily infer the conclusion.

ordinary mathematicians can typically produce an argument which establishes their premises and inferences from less controversial claims with reasonable speed and reliability[5]. Accordingly, we can think of them as having a kind of substantive implicit access to some argument which we would recognize as non-question begging. In contrast, the mathoids have no access to, or ability to derive, the additional premises from premises which we would accept[6]. Thus, even if experienced mathematicians do form beliefs on the basis of proofs that would not be immediately convincing to untrained individuals, one can argue that they only count as having knowledge in virtue of having a kind of implicit access to the kind of more explicit proof which would be accepted by the untrained.

While many philosophers share this intuition, some philosophers with strong reliablist or coherentist intuitions will feel that it is trivial that the mathoids know, because of the internal coherence and (even potentially explicable) external reliability of their beliefs. The mathoids' beliefs are clearly internally coherent and clearly the result of reliable methods in the sense of producing true beliefs at close possible worlds[7]. However, it is worth noting that if these features suffice to make a belief default reasonable it would force us to accept some extremely strange conclusions about what can be known a priori. Indeed, it would authorize even the radical conclusion

[5]Maybe there will be some cases where they accept instances of some fundamental theorem without being able to prove it from less controversial things on the spot, but in such cases it might be claimed that ordinary mathematicians' knowledge depends on their having the right kind of memory and testimony facilitated connection to some previous act of proof.

[6]They would, of course, accept each step in Wiles' proof. However, they have no better ability to construct this proof than we do ourselves.

[7]See the discussion of PEASOUP later in this paper for an example of the problem that faces more demanding notions of reliability when applied to default reasonableness.

that contingent laws of physics are knowable a priori[8]. Consider the case of the doctoroids below.

The Doctoroids Suppose that in order to save 6 years of medical education we engineer creatures who are disposed to find certain true propositions of organic chemistry and medicine immediately compelling. As soon as they consider the question of how smoking effects humans, they find it immediately obvious that it causes cancer[9]. The doctoroids feel no compulsion to produce further justification for their judgements nor have they (generally) done or heard about the kind of empirical experiments which we would normally judge to be necessary to support these conclusions. Suppose further that we don't provide them with evidence that would allow them to deduce that we tampered with their methods of reasoning in this fashion.

Just like the mathoids' mathematical intuitions, the doctoroids' scientific intuitions are true, internally coherent, the result of a reliable process and reasoning from them is a reliable way of forming true beliefs. Also, like the mathoids, they could in principle discover their origins and the reliable way in which their belief was formed. Thus, if one supposes that these features are sufficient to make a priori reasoning count as default reasonable then one must allow that the doctoroids gain a priori knowledge just as well as the mathoids. But, this would means that propositions like 'smoking causes cancer' or the chemical theory summarized in the periodic table of elements could qualify as a knowable a priori.

[8]Ultimately I don't want to claim that the doctoroids don't know. I merely mean to point out that the kinds of principles which would allow one to deduce that the mathoids know merely from the fact that they have internally coherent reliable belief forming methods are highly controversial

[9]We need not suppose that they find it obvious in phenomenologically the same way that we and the mathoids find mathematical propositions obvious. All that matters in the present contexts is that there could be creatures who believe fundamental facts about chemistry without appeal to any further potentially unreliable argument or inference.

Having evoked some sympathy for the intuition that the mathoids don't know, I will now turn to the main task of this paper which is to argue that this intuition should ultimately be rejected (at least by philosophers who accept default reasonableness approaches to the a priori). I will consider a number of ways in which one might hope to specify principled sufficient conditions for default reasonableness which are wide enough to ground paradigmatic cases of mathematical knowledge but narrow enough to avoid implying that complex truths like FLT could be known without further argument (by some creatures). I will argue that each proposal fails to live up to this challenge.

For concreteness, I will use ordinary mathematicians' acceptance of $0+1 = 1$ as an example of a default reasonable belief. Philosophers who don't think mathematicians know that $0 + 1 = 1$ but only some paraphrase of this statement are invited to substitute that paraphrase for $0 + 1 = 1$ in the argument below. Philosophers who think that we can only know that $0 + 1 = 1$ by appeal to some more basic truths are invited to substitute one of these truths for $0 + 1 = 1$[10].

3. Ground Clearing

Let me begin by doing some ground clearing. In this section, I will review some common initial suggestions that come up when people are first exposed to the challenge presented in this paper. These aren't well developed theories but seeing why our intuitive first responses fail will help motivate the more complex proposals considered latter.

[10]Note that, as we are assuming a priori knowledge is grounded in default reasonable beliefs, if we can have a priori knowledge of $0 + 1 = 1$ it must be justified via default reasonable beliefs and inferences.

3.1. **Infallibility & Inconceivability of Failure.** A first proposal arises from the idea that infallible belief forming methods qualify as default reasonable. Perhaps what makes our belief in claims like $0 + 1 = 1$ default reasonable is that it would be metaphysically impossible for the method of assuming $0+1 = 1$ to yield a false belief. This proposal immediately fails to explain why the mathoids don't know Fermat's last theorem, since assuming FLT is just as infallible a method as assuming that $0 + 1 = 1$.

A similar objection applies to the idea that what makes our mathematical assumptions justified is that it is inconceivable that making these assumptions should lead us to a false belief. If conceivable is understood in an objective fashion, such that conceivability requires metaphysical possibility, then forming a false belief by assuming FLT is just as inconceivable as forming a false belief by assuming that $0 + 1 = 1$. If, on the other hand, inconceivability is understood epistemically - as meaning that it would be epistemically justified to neglect the possibility that believing $0+1 = 1$ will lead to a false belief - then saying that it is epistemically justified to assume $0 + 1 = 1$ but not FLT merely restates the difference which needs to be explained.

3.2. **Coherence.** A second proposal says our belief in claims like $0+1 = 1$ is default reasonable because these facts form part of a coherent network of other claims within a largely empirically adequate picture of the world. Many obvious-feeling mathematical claims can also be derived from one another. For example, there are various different ways of axiomatizing arithmetic which are inter-derivable. Thus, it is plausible that $0 + 1 = 1$ can be supported by other elements in a web of mathematical claims that feel obvious to us. Perhaps this heightens our justification for believing the claim that $0 + 1 = 1$ or removes some defeater to this justification.

However, this criterion is too lax to exclude the mathoids. By hypothesis, the body of mutually illuminating theorems which the mathoids find obvious is coherent. Since this collection of theorems contains only mathematical truths, it can only conflict with sense experience to the extent that this sense experience is misleading (and, by Wiles' proof, to no greater degree than our mathematical assumptions conflict with experience). Just as above, one will be able to derive some of these theorems from others and support generalizations with particular cases. Even though the mathoids may lack Wiles' proof of FLT, they can use FLT to give alternative proofs of many (famous and useful) restricted cases of the conjecture and prove FLT itself from various other complex mathematical truths which we find equally non-obvious. Thus, their mathematical beliefs will also form part of an internally coherent and a largely empirically adequate picture of the world.

3.3. **Explicable Faculties.** A third suggestion arises from the thought that what's wrong with the mathoids is that they don't have any a satisfactory account for how they could have gotten correct mathematical intuitions. By hypothesis, the mathoids don't know that we formed their mathematical intuitions in response to reliable evidence. Thus, one might suggest that we are justified while they are not because we can tell a plausible story which explains how we came to have reliable intuitions while they cannot.

This suggestion can be preciscified in two different ways, but neither is adequate. First, one might propose that it is only default reasonable to accept claims that feel obvious about a given subject matter if you currently possesses a satisfying story about how you developed reliable intuitions with regard to this subject matter. But, this proposal is surely too strict. It's famously difficult to explain why our mathematical beliefs should have any correlation with their subject matter. Benacerraf points out in

[Benacerraf, 1973] that realists about mathematics face a serious problem in accounting for our access to mathematical facts[11]. Presumably we don't want to say that no one can count as knowing that $0 + 1 = 1$ until they come up with an adequate response to the access problem.

Alternately, one might propose that it is default reasonable to accept claims that feel obvious about a given subject matter if you could, in principle, discover how you developed reliable intuitions about this subject matter. This proposal allows us to be justified in believing that $0 + 1 = 1$ *now*, in virtue of the fact that we could some day discover an adequate explanation for the reliability of our intuitions. Unfortunately, it now fails to distinguish our mathematical assumptions from those of the mathoids. Just as we could learn how evolution (whether genetic or memetic) shaped us to have reliable intuitions about mathematics, the mathoids could learn that we shaped them to have reliable intuitions as well. As both the mathoids and normal humans plausibly require further empirical information to discover a satisfying story about how they came to have reliable intuitions, this consideration also fails to provide the needed distinction.

3.4. **Acceptability to All Thinkers.** A fourth proposal says that it is default reasonable for us to believe certain propositions because these propositions must be accepted by any being that counts as thinking. Here, the idea is that the mathoids could retreat from the degree of mathematical knowledge which they accept to some more limited body of mathematical propositions while still counting as thinkers. For example, they can retreat to the mathematical propositions which normal humans find immediately obvious. But, one might argue that we cannot retreat from our primitive

[11]Giving up realism about mathematical objects doesn't solve the problem as fictionalists face a similar problem in accounting for our ability to get true beliefs about what truths must hold in a particular fictional scenario. Similar difficulties plague many other philosophies of mathematics.

logical intuitions to question whether they work, because these claims are so fundamental that if we retreated from them we wouldn't count as reasoning at all.

Although this proposal avoids implying that the mathoids know, the sufficient condition on default reasonableness it generates is far to weak to make sense of our apparent a priori knowledge. If any propositions are 'impossible to retreat from' in the relevant sense, far too few are to provide an adequate foundation for our apparent a priori knowledge.

First, there are reasons to doubt that our sample claim $0 + 1 = 1$ must be accepted by any thinker, as well as reason to doubt that any collection of simpler truths from which $0 + 1 = 1$ can be derived has this feature. For, it seems that someone could accept first-order logic without accepting any claims that quantify over mathematical objects and still count as a thinker. Explaining our apparent knowledge of mathematical induction in this framework is even more difficult. Second, even if one were able to identify all acceptable starting points for mathematical reasoning with simple first order logical tautologies, it is not clear that the principles of classical logic themselves are impossible to retreat from in the relevant sense. Arguably, someone could accept intuitionistic logic but not the law of the excluded middle and still count as a thinker[12]. Thus, although the requirement of acceptability to all thinkers may indeed constitute a sufficient condition for being an acceptable starting point, it is far too weak to distinguish our paradigmatic cases of a priori reasoning from what the mathoids are doing.

4. Conceptual Truth

With this ground clearing done, I will now turn to some more complex stories about what can make certain beliefs default reasonable. In

[12] See the note in [Williamson, 2008].

[Peacocke, 1992, Peacocke, 2005] Christopher Peacocke proposes a sufficient condition for default reasonableness which invokes the notion of conceptual truth.

Peacocke suggests that, in order to possess certain concepts we must find certain statements and inferences involving these concepts primitively compelling. For example, one might think that someone couldn't possess the concept of the material conditional ($\supset$) without being willing to infer B from A and $A \supset B$. He then proposes that these possession conditions for a concept, the bundle of statements and inferences which a person has to accept to possess the concept, play a number of very important and related philosophical roles. First, we can individuate concepts by their possession conditions. Second, the sense of each genuine concept will systematically determine a semantic value for that concept (a reference if we are talking about the concept associated with a name) in such a way as to necessarily ensure that all the characteristic statements and inferences associated with that concept come out to be truth preserving. As Peacocke puts it, the semantic value of a concept "is fixed in such a way as to make the belief forming practices mentioned in its possession conditions always yield true beliefs and to make the inferential principles mentioned therein always truth preserving."

As a result of this metasemantic fact, doing a priori reasoning by putting together conceptually necessary statements and inferences will constitute an infallibly reliable method of forming beliefs. Peacocke further maintains that forming beliefs in the relevant conceptually necessary way is (defeasibly) justified. As a result, in cases like the example of the material conditional above, in which the relevant way amounts to finding a certain claim immediately compelling, belief in the claim is default reasonable. Call these claims (or inference rules) conceptual truths.

This view is attractive in a number of respects. It avoids the undesirable conclusion that conceptually true statements are somehow claims about acts of linguistic stipulation or language conventions. It allows us to systematically explain what goes wrong with pseudo concepts in the literature like tonk [Prior, 1960] or boche[13] [Dummett, 1991]. It also promises to dispel some intuitive worries about how to fit a priori knowledge into a broadly naturalistic and scientific picture of the world, since no special perceptual contact with concepts is required for a priori knowledge of conceptual truths. This degree of helpfulness should not be overstated, as an explanatory question still remains as to how we came to accept conceptual truths corresponding to genuine concepts instead of the invalid introduction and elimination rules associated with psuedo-concepts like tonk and boche[14], but, arguably, Peacocke's ideas about conceptual truths constitute a helpful first step.

However, I will now argue that, whether or not it is correct, Peacocke's account of how possession conditions for concepts can make certain beliefs default reasonable does not suffice to distinguish our mathematical beliefs from those of creatures who intuitively lack any adequate argument. I will discuss two problems.

First, there's a worry as to whether there are enough conceptual truths (in Peacocke's sense) to explain our a priori mathematical knowledge. If we individuate mathematical concepts in an ordinary way it appears that there will not be.

[13]Tonk is the would-be concept that has the introduction rules for or and the elimination rules for and. Boche is the would-be concept such that 'x is a boche' can be inferred from 'x is a german' and 'x is cruel' can be inferred from 'x is a boche.' Peacocke's theory explains why these inferences fail to be associated with any genuine concept, as it will not be possible for any determination function to assign semantic values to 'tonk' or 'boche' in such a way that the above indicated mandatory inferences come out true.

[14]Even if one does not need to justify the belief that one has locked onto a genuine concept to gain justification but this still leaves a significant access problem as to why we accept genuine conceptual truths and not impostors.

Second, there's a worry about concepts that 'pack in' intuitively non-obvious mathematical facts. Even if concepts can be individuated in a way that justifies our mathematical beliefs, but not those of the mathoids, Peacocke's account of conceptual truth entails that similar creatures, the mathoids*, could know FLT by means of a trivially short, intuitively unacceptable, argument.

4.1. **Individuating Concepts.** If we follow ordinary practice in attributing concepts, it's unclear whether there are any conceptual truths associated with mathematical concepts at all. The problem is that we regard people possessing a variety of different premises and inference rules as all having the concept of number. For instance, one person might understand the numbers in terms of induction (if $P(0)$ and $(\forall n)\,[P(n) \implies P(n+1)]$ then $(\forall n)P(n)$) while another person might understand the numbers in terms of the least number principle (if $(\exists x)P(x)$ then $(\exists x)\,[P(x) \wedge (\forall y < x)\neg P(y)]$). Despite accepting different inference rules, both people meet our ordinary standards for possessing the concept number (even if these individuals are unaware of the equivalence of these principles). Thus, it is doubtful whether there any particular claims which must be accepted immediately by anyone who counts thinking about the numbers[15].

It might at first appear that one could avoid this issue about alternative grips on a single mathematical concept by extending Peacocke's account as follows. Instead of requiring that *everyone* who posses a concept accept

[15]In [Williamson, 2008] Timothy Williamson advocates a much stronger form of the objection I have outlined above. He claims that there are no statements which must be accepted by anyone who understands a particular concept, much less propositions which are conceptual truths in the more exacting sense at issue here. He gives examples like philosophers who doubt that all vixens are foxes because they think all apparent vixens are cleverly painted dogs and accept an unusual account of the universal conditional which renders universal quantification over empty domains to be false. I think this is an interesting argument, however, I want to stress that my point above does not depend on accepting anything so strong as Williamson's more general claim.

a claim for it to qualify as a conceptual truth we might instead demand only that everyone possessing the concept must accept *some* premises and inference methods from which the claim can be derived. Adopting this proposal would allow the least number principle to count as a conceptual truth despite the existence of people who understand the numbers in terms of induction. Though someone who understood the concept of numbers by way of induction might fail to believe the least number principle, the least number principle is derivable from the premises and inference rules they do accept[16]. However, as Wiles' proof demonstrates, FLT is provable from premises that we accept about the numbers[17]. Thus, if this version of the conceptual truth theory justifies our beliefs about the numbers it also justifies the mathoids belief in FLT.

Perhaps, this unwelcome consequence can be avoided by appealing to some notion of what can be established via a *short* argument. We might say that, in order for a claim to qualify as a conceptual truth, everyone who understands the relevant concept must accept some premises and inference rules from which that claim is derivable via a short argument. This would allow us to distinguish the proof of induction from the least number principle and the proof of Fermat's last theorem. However, it is hard to see how the notion of short argument (and the induced individuation of concepts) can be grounded in any thing more principled than contingencies of human psychology. In contrast, it is easy enough to see how psychological facts about the speed of typical human reasoning could have lead us to cluster together inference bundles which are derivable from one another via short arguments under a single concept. Normal human speeds of making inferences allow

[16]Induction and the least number principle can easily be seen to be equivalent over a weak base theory.

[17]If you doubt that FLT is provable without set theoretic principles which are not conceptual truths about the numbers then substitute some other deep claim about the numbers which has only long proofs from standard arithmetic axioms.

us to treat people who find certain slightly different bundles of inferences immediately compelling as deploying the same concepts that we do, e.g., talking and thinking about the numbers, because we can always turn the kinds of arguments which they find compelling into the kinds of arguments which we find compelling without taking much time or trouble. However, on this view, if we were smarter we would individuate concepts more broadly. Thus, we would allow multiple inference bundles whose equivalence can only be established by a longer argument to all qualify as conceptual truths for that concept. As a result, the distinction between conceptual truths and non-conceptual truths will ultimately reflect contingent psychological facts about how quickly humans can derive the premises of some inference bundles from others. In contrast, if one wanted to spell this proposal out in a non-psychologistic way, one would need to appeal to something other than facts about what people can easily and reliably prove to distinguish short arguments.

Alternately, we might give up intuitive verdicts about concept possession, and say that there are genuinely different concepts corresponding to each of the different grips on the notion of number described above. Saying this involves some bullet-biting as it seems quite possible that actual people will acquire the concept of number in the slightly different ways mentioned above. Thus, people will express different propositions when they say things like "For all natural numbers a and b , $a + b = b + a$."

4.2. **Interaction of Conceptual Truths.** Even if we take for granted some workable scheme for individuating concepts in a way that makes sufficiently many claims turn out to be conceptual truths, we face a second objection. Although the conceptual truth proposal lets us deny that the mathoids know FLT immediately, allowing that all conceptual truths are default reasonable forces us to accept that other creatures (the mathoids*) could know FLT on the basis of an equally unconvincing argument.

The problem is that, as Boghossian has pointed out [Boghossian, 2003], theories which say it is default reasonable to accept any genuine conceptual truth give rise to problem-cases with regard to concepts that seem to 'pack too much in.' The conceptual truth proposal authorizes us to reason about any coherent concept, not merely those we are justified in believing are coherent. But, this permission to use non-obviously coherent concepts can be parlayed into permission to explicitly believe correspondingly non-obvious necessary truths via a short argument. In particular, we can design a concept whose coherence depends on the truth of FLT and use that concept to infer the truth of FLT in the numbers.

To illustrate this point, consider the concept 'schnumber' characterized by the following bundle of claims:

(1) The schnumbers satisfy the Peano axioms for arithmetic.
(2) No proper initial segment of the schnumbers satisfies the Peano Axioms.
(3) The schnumbers satisfy FLT.
    - i.e., There are no schnumbers a, b, c, n with $n > 2, a, b, c > 0$ such that $a^n + b^n = c^n$

Since FLT is true of the numbers, the above claims characterize a coherent concept as one can assign the same extension to the schnumbers as one does to the numbers. Thus, if the mathoids* assume these statements and thereby possess the concept of schnumber, then belief in these statements is default reasonable for the mathoids*. But, if they are entitled to these premises then they can give a quick 'proof' that Fermat's last theorem holds for the numbers as follows:

Theorem 4.1. *There are no natural numbers* a, b, c, n *with* $n > 2, a, b, c > 0$ *such that* $a^n + b^n = c^n$

Proof. By standard results in mathematical logic, any structure satisfying the Peano axioms has an initial segment isomorphic to the numbers. By 2 that initial segment can't be proper and hence the schnumbers are isomorphic to the numbers. By 3 the schnumbers satisfy FLT and by isomorphism so do the numbers. □

Thus, it would seem that any creatures with the schnumber concept can come to know that FLT is true by way of the short argument above. Yet, intuitively, the argument above is just as inadequate as the mathoids' original one line proof[18].

Now what can be done to respond to this worry?

Within the limits of Peacocke's view, we may attempt to block the proposal above by denying that there's a genuine concept schnumber corresponding to the bundle of inferences above. One might argue that, although coherent, the schnumber concept is bad because it 'packs something extra in.' Thus, one might try to say that there are only genuine concepts corresponding to bundles of inferences such that none of the relevant premises or

[18]Boghossian [Boghossian, 2003] argues against Peacocke by considering the concept flurg which has an introduction rule allowing one to infer that 'x is a flurg' from 'x is an elliptical equation' and an elimination rule allowing of to infer that 'x can be correlated with a modular form' from 'x is a flurg.' While also providing necessarily truth preserving inferences one might object that flurg is not truly a concept in the appropriate sense. After all, we do not typically introduce concepts by directly specifying introduction and elimination rules and one might feel there is something suspect about defining a concept in terms of inferences about some other concept. In contrast, the concept schnumber is characterized in exactly the same way as intuitively acceptable concepts like number. Indeed, if the conceptual truth proposal is to justify our mathematical beliefs it must justify our belief in claims 1 and 2 for the numbers. Moreover, if the proposal is to account for the standard mathematical practice of applying results in one area of mathematics to another, e.g., the use of analysis in number theory, it must authorize the sort of application used in the argument above. Thus, it would seem that the conceptual truth proposal is unable to provide the desired distinction between intuitively acceptable and unacceptable arguments.

inference rules are redundant: no premise or inference rule can be derived from some combination of the other constitutive premises and inference rules associated with that concept.

One problem with this line of response is that it's not clear that there is any psychologically realistic way to individuate the constitutive premises and inference rules for our mathematical reasoning which satisfies this constraint. The premises and inferences which we find immediately obvious and take as unargued premises in apparently justified mathematical reasoning seem to involve a great deal of redundancy. For instance, the least number principle and the principle of induction both seem obvious and can figure in apparently justified deductions about the integers.

More importantly however, it is trivial to modify the schnumbers concept so it's not redundant. Rather than asserting that "The schnumbers satisfy the Peano axioms for arithmetic." we simply modify the first claim to instead assert that "If FLT is true in the schnumbers then the schnumbers satisfy the Peano axioms for arithmetic." On this modified description we can't remove either claim without substantially changing the concept under consideration.

Thus, I do not think Peacocke's metasemantic approach to a priori knowledge ultimately gives us the resources to distinguish paradigmatically good mathematical reasoning and intuitively unsatisfying mathematical reasoning. If we think of concepts as inference bundles (as Peacocke seems to) then very powerful and substantive assumptions can be packed into a concept. As Peacocke doesn't provide any means to determine which inference bundles correspond to genuine concepts, his approach fails to provide the principled distinction we are looking for between the assumptions made by the mathoids and those we make.

Going outside of Peacocke's own story, we may note that when Boghossian first drew attention to this overgeneration problem he proposed that the problem could be solved by maintaining that it is only default reasonable to assume conceptual truths associated what might be called 'minimal' concepts. Thus, he thinks that we are justified in thinking things like 'if there are numbers they have the various features listed in the Peano axioms.' But, we are not justified in accepting the Peano axioms outright. This view has it's attractions but, we should note that it cannot help with the task at hand.

If one takes this approach then one will need to appeal to something other than default reasonableness in virtue of conceptual truth to explain our mathematical knowledge. The problem is that we frequently apply results from one area of mathematics to another in a non-hypothetical fashion. When we use results from analysis or group theory to prove theorems about the numbers we don't conclude that *if* there are groups/reals then the numbers satisfy the theorem. Rather, we conclude the theorem is true. Thus, as Michael Potter has emphasized [Potter, 2007], whatever our knowledge of the numbers amounts to, it includes at least the claim that the axioms of number theory are first order logically consistent. But, this claim does not in any way follow from the kind of conditional claims about what objects must be like if there are numbers, which Boghossian is willing to allow as conceptual truths. Thus, this view no longer answers the challenge at hand providing a sufficient condition on default reasonableness which explains our a priori knowledge of mathematics but not that of the mathoids.

5. The Indispensability Approach

A very different approach to distinguishing our epistemic situation from the mathoids' arises from appeal to pragmatic considerations associated with epistemically mandatory projects. In their recent paper *How are basic belief forming methods justified* [Schecter and Enoch, 2008] Enoch and Schechter make the following proposal:

> A thinker is prima facie epistemically justified in employing a belief-forming method as basic if there is a project that is rationally required for the thinker such that:
>
> (1) it is possible for the thinker to successfully engage in the project by employing the method and
>
> (2) it is impossible for the thinker to successfully engage in the project if the method is ineffective.
>
> Moreover, where clauses (1) and (2) apply, it is in virtue of these facts that the thinker is so justified.

The idea here is that we face certain non-optional cognitive projects including,"the project of understanding and explaining the world around us" [Schecter and Enoch, 2008]. These projects are non-optional in the sense that an agent is rationally required to try to engage in them. For example, Enoch and Schechter claim that the project of understanding and explaining the world is non-optional in the sense that, "a thinker who does not inquire about the world around him is intuitively doing something wrong." Furthermore, certain methods of reasoning which allow us to successfully engage in these (supposedly) non-optional projects have the important further feature that they will succeed in these projects if any method can.

Enoch and Schechter want to suggest that this feature gives us a kind of prima facie epistemic warrant for using these methods. We are rationally required to engage in certain cognitive projects and using these methods will let us succeed if anything will. A natural temptation is to read Enoch and Schechter's proposal as giving an account of when it's ok (in some sense) to make epistemically unjustified assumptions. However, Enoch and

Schechter explicitly reject this interpretation and make it clear that, on their proposal, beliefs formed by a method that satisfies (1) and (2) thereby have (defeasable) epistemic justification.

Can appeals to this kind of indispensability distinguish our methods of mathematical reasoning from those of the mathoids?

Insofar as Enoch and Schechter say very little about what projects are non-optional or what constitutes success in these projects, it is hard to definitively evaluate whether they can distinguish our mathematical practice and that of the mathoids. However I think there are strong prima facie reasons for pessimism. For example, if Enoch and Schechter say that learning about mathematics is part of a non-optional cognitive project then their criteria can't distinguish our methods from those of the mathoids[19]. Both our methods and the mathoids' will satisfy the first condition since both are reliable ways of learning about mathematical facts. As the mathoids can prove exactly the same results as we can, their methods succeed in exactly the same scenarios as ours do. Thus, condition (2) ('it is impossible for the thinker to successfully engage in the project if the method is ineffective') is satisfied for the mathoids if we satisfy it. Hence, Enoch and Schechter's criteria fail to distinguish the mathoids' methods from our own.

Additionally I will argue that Enoch and Schechter's proposal already faces serious internal problems with regard to the single motivating example they do provide details about: the task of vindicating inference to the best explanation (IBE). In order to vindicate the default reasonableness of IBE in the way that they claim, Enoch and Schechter need to show that IBE satisfies their second requirement, in particular, that it is impossible to successfully

[19]Here I am taking 'our methods' to specify the specific content of the assumptions and inferences that can be made. If, instead one thinks of our method as something like 'believe the mathematical claims that seem obvious to you' then the mathoids and ourselves are employing the same method, so Enoch and Schechter's account vindicates one if it vindicates the other.

engage in the project of understanding and explaining the world around us if IBE is ineffective. This is presumably what distinguishes IBE from other methods of understanding and explaining the world around us which are intuitively not justified, such as assuming the true laws of fundamental physics.

But how does IBE itself fare with regard to this requirement? Consider worlds that satisfy the constraint PEASOUP indicated below.

PEASOUP Everything outside a 5 foot radius around you is pea soup, but when you walk by the soup forms up into ordinary material objects around you (including the photons that mediate vision), making it appear as if the normal laws of physics govern the entire world.

With this in mind, let PEASOUPISH IBE be the method of reasoning which resembles inference to the best explanation but assigns very high prior probability (perhaps 1) to the hypothesis that the world obeys elegant laws up to a radius of 5 feet and is composed of PEASOUP outside that radius.

Now, presumably, we wouldn't count IBE as a successful method for understanding and explaining the world if PEASOUP was actually true[20]. But, this raises a serious problem for the claim that worlds where IBE fails are worlds where no other method would succeed. For, it seems that PEASOUPISH IBE is precisely a method that succeeds at some of these worlds where IBE fails.

[20]If Enoch and Schechter claim that IBE still succeeds at PEASOUP worlds merely on the basis of accurately predicting the behavior of objects in a 5 foot bubble around the observer then we must apply the same criteria to assess the success of PEASOUPISH IBE. However, at any world in which IBE succeeds in predicting the behavior of nearby objects so does PEASOUPISH IBE. Thus, if we are to count IBE as succeeding at PEASOUP worlds then we must count PEASOUPISH IBE as succeeding at the actual world and all other worlds where IBE makes accurate predictions about nearby objects. More generally it becomes unclear why we should think that IBE succeeds at any worlds where PEASOUPISH IBE. If it does not then PEASOUPISH IBE will come out to satisfy Enouch and Schecter's second criteria if IBE does. This is an unwelcome result because although Enouch and Schecter are happy to allow that slight variants on inference to the best explanation are justified, they would presumably reject the conclusion that someone can be justified in believing a priori that everything outside of a 5 foot radius is pea soup.

How might Enoch and Schechter respond to this challenge? One possible line of response concerns the issue of what qualifies as a belief forming method. In a footnote Enoch and Schechter appeal to an intuitive sense in which not all "arbitrarily complex functions from beliefs (and other mental states) to beliefs" count as genuine belief forming methods. Unfortunately, they offer no other guidance as to what qualifies as a genuine belief forming method. Perhaps Enoch and Schechter would try to save their justification of IBE's default reasonable status by arguing that PEASOUPISH IBE doesn't count as a genuine method of forming beliefs.

However, it is hard to see how this restriction can be motivated by anything in the intuitive notion of belief forming method which Enoch and Schechter appeal to. We might intuitively require that belief forming methods have to be general in some way: they should yield verdicts about a range of different cases, and these verdicts should be produced by some kind of uniform process. Thus, arguably, the method of just making some short list of assumptions and stopping there does not count as a genuine method in the intuitive sense. However, PEASOUPISH IBE seems to be a quite unified and general belief forming method, certainly no less acceptable than rejecting extreme skepticism and then applying IBE.

Perhaps Enoch and Schechter could respond to this criticism by stipulatively defining a more restrictive notion of what counts as a genuine method. However, in order to deny that PEASOUPISH IBE constitutes a genuine method, Enoch and Schechter would have to provide some *principled* sense in which IBE (which they are claiming does constitute a genuine method) qualifies but mere Bayesian updating on certain priors as per PEASOUPISH IBE does not. Thus, they would have to provide a distinctively unified and elegant characterization of the good priors associated with IBE or show that IBE ought to be understood in some more elegant and uniform way than Bayesian updating on priors. However, the project of providing a unified logic of induction is infamously hard. Even if they had such a unified logic in hand, Enoch and Schechter would still have to demonstrate that no comparably elegant method yields true beliefs in a PEASOUP world. Unfortunately, their article provides no hint as to how either of these things might be accomplished.

Alternatively, we might try to avoid the problem above by weakening Enoch and Schechter's requirement that it be literally **impossible** for the thinker to successfully engage in the relevant project if the method is ineffective. Perhaps, there is something epistemically important about the fact that IBE is only bested by alternative methods like PEASOUPISH IBE at quite remote possible worlds.

One way of fleshing out this idea is to restrict the scope of the claim above to some range of worlds which are sufficiently close to the thinker. However, this modification seems to entail that accepting the actual laws of physics, or at least the regularities embodied by the periodic table of elements is reliable. One might also worry about whether one can provide a provide a principled way of drawing the boundary between worlds which are and are not sufficiently close.

Another way of fleshing out this idea which avoids the last worry above would be to say that success (at the relevant cognitive project) has to be impossible *at the closest worlds where the method is unreliable.* This would save IBE if it were plausible that the closest possible worlds where IBE fails are completely anarchic ones where no belief method would succeed. Unfortunately, though this seems highly contentious at best. For the actual physical laws of our world would seem to allow the construction of brains in vats. Thus, there would seem to be some centered possible worlds which are quite close to the actual one where the center is a brain in a vat and doing induction the way that we ordinarily do it will lead to beliefs which are at least as false as those produced by PEASOUPISH IBE. Yet, it would seem that there are other methods which could work reasonably well in these worlds. The method of (so to speak) 'BIV IBE' where one assumes that one is a brain in a vat created by creatures in a world with an elegant non-vatlike physics would seem to do reasonably well. Someone who had the advantage of knowing that you are a brain in a vat created by creatures largely like ourselves could usefully try a range of special strategies to learn things about the world outside the vat. For instance, they might try to communicate with the people running the simulation and request to be let out, infer things about how the simulating computer must function in light of universal computational limitations or repeatedly speak about the ethical considerations of keeping brains in vats against their will. Thus, it would seem that there are quite close worlds where IBE fails and some alternative strategy succeeds.

Thus, as it stands Enoch and Schechter's proposal can only succeed at explaining even our knowledge of their central example with some very substantial, and uncertain, assumptions or modifications and even then it fails to distinguish the assumptions we make in our mathematical arguments from those made by the mathoids.

6. Conclusion

After these negative results, let me conclude by considering what the philosophical landscape would look like if we gave up the search for a principled sufficient condition on default reasonable beliefs which can distinguish our mathematical reasoning from that of the mathoids. There are three possible ways of going forward if one gives up this hope.

First, one can draw the conclusion that a priori knowledge is not best understood solely in terms of default reasonable beliefs. Thus, for example, we might take a coherentist approach to justification and allow virtuous circles of justification. Or we might accept that some cases of a priori knowledge are justified by a special experience of rational insight. Note, however, that while this paper focuses on problems for attempts to ground a priori knowledge in default reasonable beliefs, similar problems appear if one tries to understand our justification for accepting the starting points of our mathematical reasoning in coherentist or rationalist terms. For example, we saw in section 3 that mere appeals to coherence do not suffice to differentiate us from the mathoids. Similarly, merely invoking rational intuition does not suffice to dispel the problem. If one thinks about the experiences of rational intuition as some kind of phenomenological halo (a mere feeling of obviousness and confidence) it also fails to distinguish us from the mathoids[21].

[21]One might instead prefer to think about of rational intuition as involving something more substantive. For instance, reliable use of mental images whose structure resembles that of the mathematical objects under consideration like that proposed by Chudnoff [Chudnoff, 2012b, Chudnoff, 2012a]. However, I do not think that this requirement will suffice to ensure that knowledge of a mathematical claim requires having anything that we would be inclined to recognize as an adequate argument for the claim. For, the question of whether a given mental picture is similar in structure to the mathematical objects which it is being used to represent depends on complex and non-obvious mathematical facts, e.g., a given picture might correctly represent the numbers just if FLT is true. Thus, there will be some mental pictures whose structural similarity to the mathematical claims under consideration is itself a highly non-obvious mathematical fact. Creatures who are innately inclined to use these powerful and non-obviously accurate methods of picturing will be able to know claims like Fermat's last theorem immediately by appeal to these mental pictures. Yet, even if we could some how look into their heads and see the mental pictures they are using, we would not be convinced that the relevant mathematical facts follow from the corresponding claims about the mental pictures. After all, seeing that the picture accurately represents the mathematical subject matter could even require knowing the very claim at issue. Thus, they will know without being able to provide anything that would strike us as an intuitively adequate argument.
In saying all this I do not mean to assert that no adequate story can be told about how a theory of rational insight could authorize only intuitively adequate a priori arguments, only that there is a prima face question about how this can be done.

Second, we could deny that the mathoids' assumptions are default reasonable, but say that there is no there is no *principled* feature which distinguishes their relationship to the propositions they assume from our relationship to the propositions we assume. For example, if we were willing to accept a certain kind of psychologism about default reasonableness, we might identify the default reasonable claims with the true (or necessarily true) propositions which *actual human beings* tend to find immediately obvious. On this view the property of being a priori knowable would turn out to be something like the property of being edible or jumpable. Although it can apply to items in possible worlds where there are no people at all, it sorts these items in virtue of their relationship to facts about what people can eat and jump in the actual world. Alternately, we might use psychological constraints to revive the conceptual truth approach by limiting either the kinds of concepts allowed or what applications of these concepts are acceptable. It is interesting to note that Dogramaci [Dogramaci, 2012] has independently advocated a view along these lines, where the facts about what is required for justification are somewhat arbitrary, but we enforce conformity in epistemic practices because this conformity is epistemically useful.

Third, we could simply accept that the mathoids assumptions *are* default reasonable. One might say that all necessary truths are knowable a priori by appropriate creatures - or even that all 'counterfactually robust' truths[22] are knowable a priori. One might even allow that any true proposition can be the object of default reasonable belief if they feel obvious and they are part of a suitable internally coherent cluster of beliefs.

Allowing that the mathoids know requires substantially deflating the apparent philosophical significance of our intuitive judgements about what is

[22]Truths that hold at all close possible worlds.

and is not an adequate argument. It is hard to avoid classing arguments as adequate or inadequate to justify belief. And it is tempting to think that in doing this we are tracking some general feature of general epistemic significance. If the mathoids do count as knowing then we must admit that our intuitions do not reflect general constraints on what claims can be known without further argument by all rational beings.

However, I think there are ultimately good reasons to be suspicious of this intuition that our feelings about which necessary truths are obvious enough to figure as unargued premises in a proof track a general epistemically valuable distinction. In a given context of discussion, a good argument is one that establishes its conclusion on the basis of premises that one's interlocutor believes, and inferences which they accept. It seems plausible that our notion of an adequate argument full stop is an idealization of this contextual notion based on dispositions and mental abilities shared by normal humans. Thus, on this view, our intuition that the mathoids don't know merely reflects the fact that the assumptions they make are far removed from those which a normal human interlocutor might accept.

I won't attempt to comment on which of the lines of response mentioned above are most promising. But, any response one takes forces one to confront the tension between intuitive hesitance to say that the mathoids know and the desire to provide a principled account of what allows us to gain a priori knowledge of mathematics.

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