Defending the Quantifier Variance Explanation for Mathematicians' Freedom

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February 13, 2017

Abstract

In this paper I argue that both realists and antirealists about metaontology alike can give a principled and satisfying explanation of mathematicians' apparent freedom to introduce new kinds of mathematical objects by invoking Quantifier Variance. I will note that taking this view about normal mathematical assertions lets one avoid known problems for popular philosophies of math. I will then address some common objections to Quantifier Variance (as they apply to my proposal), arguing that one need not accept any controversial claims about the analyticity of mathematical existence claims in order to deploy my proposal, and addressing worries that –if successful– it would 'prove too much.'

1 Introduction

Philosophers of mathematics have been much struck by mathematicians' apparent freedom to introduce new kinds of mathematical objects, such as complex numbers, sets and the objects and arrows of category theory. Accordingly, a range of different philosophies of mathematics (from platonist to nominalist ones) provide some way to make sense of this freedom in a truth-value realist fashion. However, existing proposals have run into serious problems of various kinds. In this paper I will defend giving a Quantifier Variantist explanation for mathematicians' freedom along the following lines.

> **Crude Quantifier Variantist Explanation:** When mathematicians (or scientists or sociologists) introduce logically coherent hy-

> potheses characterizing new types of objects, this choice can simultaneously give meaning to newly coined predicate symbols and names and change the meaning of expressions like "there is", in such a way as to ensure the truth of the relevant hypothesis. Thus, for example, mathematicians' introduction of the complex numbers might change the meaning of our quantifiers so as to make the sentence "there is a number which is the square root of -1" go from expressing a falsehood to expressing a truth.

I will argue that this realist story about mathematical objects is compatible with (and indeed motivated from the point of view of) both realism and antirealism about the project of ontology. And I will note that it avoids major worries for other popular accounts of mathematical object-talk (ranging from neologicism and set theoretic foundationalism to nominalist views like Hellman's modal structuralism).

But perhaps the Quantifier Variance explanation faces worries of its own? After articulating my preferred (modest) formulation of the Quantifier Variance Explanation, I will devote the bulk of this paper to answering some natural objections to it.

First, I will show that one need not accept any controversial claims about the analyticity of mathematical existence claims in order to give a Quantifier Variance Explanation. Then I will respond at length to ubiquitous worries that –if successful– Quantifier Variance explanations of mathematicians' freedom would somehow 'prove too much', either by making it impossible to be wrong in the ideal limit of scientific-mathematical inquiry, or implausibly implying that theologians' or yetti hunters' existence claims are guaranteed to express truths.

2 The Problem of Mathematicians' Freedom

2.1 Mathematicians' Freedom: General Motivation

So let us begin with the phenomena of mathematicians' (apparent) freedom. As noted above, contemporary mathematical practice seems to allow mathematicians significant freedom to introduce new kinds of mathematical objects, such as complex numbers, sets and the objects and arrows of category theory. For example, in a recent paper Julian Cole writes, "Reflecting on my experiences as a research mathematician, three things stand out. First, the frequency and intellectual ease with which I endorsed existential pure mathematical statements and referred to mathematical entities. Second, the freedom I felt I had to introduce a new mathematical theory whose variables ranged over any mathematical entities I wished, provided it served a legitimate mathematical purpose. And third, the authority I felt I had to engage in both types of activities. Most mathematicians will recognize these features of their everyday mathematical lives."[4].

Similar ideas are ubiquitous in modern mathematics. For example, in a celebrated essay about math education, mathematician-turned highschool teacher Paul Lockheart picturesquely evokes mathematics' combination of creative freedom and objective investigation as follows, "in mathematics... things are what you want them to be. You have endless choices; there is no reality to get in your way. On the other hand, once you have made your choices (for example I might choose to make my triangle symmetrical, or not) then your new creations do what they do, whether you like it or not. This is the amazing thing about making imaginary patterns: they talk back! The triangle takes up a certain amount of its box, and I don't have any control over what that amount is. There is a number out there, maybe it's two-thirds, maybe it isn't, but I don't get to say what it is. I have to find out what it is."[16]

And in a historical chapter in [11] Philip Kitcher notes that mathematical debates about the existence of imaginary numbers ended with acceptance that (something like) a mere demonstration of the *logical coherence* of imaginary numbers – provided by showing that pairs of reals provided a model of the imaginary numbers – made it permissible for mathematicians to quantify over them and reason in terms of them.

2.2 Problems for existing approaches

I will now discuss some problems that arise for existing (non-Quantifier Variantist) philosophies of mathematics when they attempt to explain mathematicians' freedom to introduce new kinds of mathematical objects. Ideally we'd like our general philosophy of mathematics to account for this freedom in a way that

1. avoids arbitrariness and ruling out intuitively acceptable mathematical practices
2. captures the metaphysical necessity of mathematical truths
3. honors the apparent similarity between mathematical and other ordinary language existence claims, e.g., the similarity in inferential role between 'there are numbers' and 'there are cities'.

However, popular non-Quantifier Variance approaches can mostly be divided up into a few families, which run into trouble in honoring the above desiderata as follows.

Plenetudinous approaches interpret mathematicians as talking about a large but fixed universe of mathematical objects. They explain mathematicians' introduce new mathematical structures by saying that mathematical objects are plentiful and diverse, but reject the claim that objects corresponding to all coherent stipulations exist. Instead, they posit generous ‘limits of abstraction’ such that all *acceptable* characterizations of mathematical structures can be understood as truly describing (portions of) a single mathematical universe. Thus, for example, in the case of standard set theoretic foundationalism, acceptable stipulations will be those which have a standard model in the hierarchy of sets. Plenetudinists then argue that all such acceptable mathematical stipulations will express truths because they truly describe suitable portions of a plentiful mathematical universe.

Plenetudinous approaches have trouble satisfying the first desideratum: their choice of *which* limits to impose can seem unmotivated. For example, in the case of standard set theoretic foundationalism, this worry takes the following form: if the hierarchy of sets has some definite height, why doesn’t the mathematical structure one would get by adding a layer of classes to this hierarchy (so that there are now objects corresponding all possible collections of elements from the original structure) constitute an acceptable object for mathematical investigation[1]?

To see how this problem arises more generally, note that Plenetudinists cannot say that all coherent patterns of relationships between objects are realized in some *portion* of the mathematical universe – so that all logically coherent structures will be realized somewhere within the total mathematical universe. For, intuitively, we can make sense of the notion of all possible ways of choosing objects out of a collection. And given any structure which we take *the total mathematical universe* to have, it appears that it would be possible and coherent for there to be a different structure corresponding to what you would get by

[1]Combining set theoretic foundationalism with a **potentialist** approach to the hierarchy of sets (which denies that the hierarchy of sets has any definite structure and cashes out mathematical existence claims in terms of logically possible extendability) [9][18] would let one resist this argument. This approach faces problems with the second desiderata (like those which beset the hypotheticalist approach described below), as it reduces all mathematical object existence claims to claims about sets but then holds that sets do not function like ordinary object claims and need to be specially paraphrased.

adding a layer of classes to the original universe (i.e., adding a layer of objects which 'witnesses' all possible ways of choosing from the original objects). By Cantor's diagonal argument, this structure would be strictly larger than the original one. Thus it appears there is a way some abstract objects could be related to one another which requires the existence of too many objects to be realized by any portion of (what we have supposed is) the total mathematical universe[2]

Hypotheticalist approaches like Modal Structuralism hold that the true logical form of a mathematical utterance 'ϕ' is something like a conditional claim 'if D then ϕ', where D combines all the descriptions of intended structures of mathematical objects currently in play. Thus, they take apparent quantification over mathematical objects to really involve a nominalistically acceptable claim about what is logically possible or what would have to be the case if certain foundational claims about mathematical objects were true. Hypotheticalists face problems with the third desideratum above: taking mathematical existence claims to have such a different logical structure and semantics and from existence claims about ordinary and scientific objects seems ad hoc and (ceterus paribus) unattractive.

As Stanford Encyclopedia puts it[15], "the language of mathematics strongly appears to have the same semantic structure as ordinary non-mathematical language... the following two sentences appear to have the same simple semantic structure of a predicate being ascribed to a subject:

[2]Admittedly, the above problem would not arise if we supposed that mathematical structures could only be characterized by first order logical sentences. One can have a total mathematical universe which contains subregions which corresponding to to all consistent collections of first order logical sentences. Indeed, by Gödel's completeness theorem any consistent collection of such axioms has a finite or countably infinite model, so any infinite mathematical universe will do this job[8]. However, since the incompleteness theorems tell us that it is impossible to express our conception of paradigmatic mathematical structures like the natural numbers by any finite or r.e. collection of first order logical axioms, we cannot make this assumption that only first order logical descriptions of putative structures will/can be considered by mathematicians.

(4) Evelyn is prim.

(5) Eleven is prime.

This appearance is also borne out by the standard semantic analyses proposed by linguists and semanticists."[3]. Thus, saying that existence claims about mathematical objects actually have a different logical form than claims about people or other objects can seem ad hoc and unmotivated. An analogous worry can also be raised for **fictionalist** approaches. Since claims that numbers exist and cities exist seem to have the same logical role and status in the minds of ordinary speakers, it can seem ad hoc to say that these speakers are unknowingly participating in a fiction in the former case but not the latter.

3 Quantifier Variance and how it promises to help in philosophy of mathematics

With these problems for alternative ways of explaining mathematicians' freedom in mind, I will now state my preferred version of the Quantifier Variance thesis and then show how we can use it to provide an attractive truth value realist (and modestly ontologically realist) explanation for this phenomenon.

3.1 The Core Quantifier Variance Thesis

To motivate Quantifier Variance, imagine speakers of a language that only quantifies over chunks of matter, who then begin 'talking in terms of' extra things like shadows and holes (which do not appear to be identifiable with any mereological fusions of particles). It is attractive to think that such people would wind up expressing truths in the same way that speakers of our language do. Such a change in language would seem to involve a shift in the meaning of some logi-

[3]This example comes from [2] pg. 288, but the point goes back to [1].

cal vocabulary since, for example, it could change the truthvalue of the Fregean sentence which says 'There are n things' using only first order logical expressions and equality[4].

But there doesn't seem to be anything deeply scientifically motivated or metaphysically special about describing the world in terms of holes and shadows. So it seems like further linguistic changes to our language which get us to 'talk in terms of' even more new kinds of objects should be possible. To put this point more forcefully, note that our ability to speak the truth by talking in terms of holes and shadows doesn't seem to depend on anything like the scientific indispensability of holes and shadows. And (intuitively) Occam's razor doesn't motivate us to deny the existence of such objects, even if no explanatory project forces us to posit them. Rather, it suffices that we have somehow managed to fix on (something like) a logically coherent criterion for how many holes and shadows are supposed to exist in each Lewissian metaphysically possible world and what other properties these objects are supposed to have.

Thus, it seems that we could introduce new observation practices (like those for holes and shadows) and start talking in terms of *these new kinds of* objects as well. Note that when we change our language in this way we are not creating these objects. The existence of holes and shadows is not caused by or grounded in the existence of language users who talk in terms of holes and shadows, and it will be true to say "there were holes before there were people, and before I started talking in terms of them." Instead we are merely changing our language so that some sentences, e.g., "there is something in the region of the flagpole which is not made of matter" go from expressing a false proposition in our old language to expressing a different, true, proposition in our current language[5].

Also note that Quantifier Variance holds that we can coherently think and

[4]For example the sentence that says there are two things $(\exists x)(\exists y)[\neg x = y \wedge (\forall z) z = x \vee z = y]$.

[5]See [5] for a vigorous development of this point which I found very helpful.

talk about languages more ontologically profligate than our own *without* employing or presuming the existence of some most natural and most generous quantifier sense which different possible languages are all quantifier restrictions of[6] some maximal and most natural quantifier sense.

I will call the idea that there are many variant senses of the quantifier corresponding to more and less generous notions of existence the **Core Quantifier Variance Thesis**. It states that that that the "$\exists$" symbol can take on a range of different meanings which are all (somehow) existential quantifier-like as follows[7].

> **Core Quantifier Variance Thesis**: The English word 'exists' takes on a range of meanings[8] in different contexts, such that
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> - all these variant meanings satisfy the usual first order syntactic inference rules associated with the existential quantifier[9]
> - it is not the case that all these variant meanings must be understood as quantifier restrictions of a fundamental most generous sense of the quantifier[10].

[6] In REDACTED I outline how one can non-paradoxically discuss languages that allow for a more generous interpretation of existence than used in one's current language.

[7] This definition is heavily influenced by Sider e.g. [20]

[8] For ease of exposition, I will usually talk about shifts in mathematical context as giving rise to shifts in the sense or meaning of the quantifier. However, I don't mean to rule out the possibility that some alternative theory could be given on which the meaning of the quantifier symbol always stays the same while its contribution to the truth conditions for sentences shifts.

[9] Specifically "($\exists I$) If $\Gamma \vdash \theta$, then $\Gamma \vdash \exists v \theta'$, where θ' is obtained from θ by substituting the variable v for zero or more occurrences of a term t, provided that (1) if t is a variable, then all of the replaced occurrences of t are free in θ, and (2) all of the substituted occurrences of v are free in θ'." and "($\exists$E) If $\Gamma_1 \vdash \exists v \theta$ and $\Gamma_2, \theta \vdash \phi$, then $\Gamma_1, \Gamma_2 \vdash \phi$, provided that v does not occur free in θ, nor in any member of Γ_2."[19]

[10] The purpose of this second clause above simply to distinguish the kind of multiplicity required by Quantifier Variance from the bland claim that we sometimes speak with implicit quantifier restrictions, as when we say 'All the beers are in the fridge'.

3.2 How Quantifier Variance Helps

Invoking the Core Quantifier Variance Thesis (that non-metaphysicians at least have substantial freedom to deploy variant senses of the quantifier which are not mere quantifier restriction of some maximally natural fundamental notion of existence) lets us explain mathematicians' freedom to introduce new kinds of apparently coherent objects along the following lines.

> **Quantifier Variance Explanation of Mathematicians' Freedom**: When mathematicians (or scientists or sociologists) introduce coherent hypotheses characterizing new types of objects, this choice can behave like an act of stipulative definition, which not only gives meaning to newly coined predicate symbols and names but can change/expand the of meaning expressions like "there is", in such a way as to ensure the truth of the relevant hypothesis.
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> Thus, for example, mathematicians' acceptance of existence assertions about complex numbers might change the meaning of our quantifiers so as to make the sentence "there is a number which is the square root of -1" go from expressing a falsehood to expressing a truth. Similarly, sociologists' acceptance of ontologically inflationary conditionals like, "Whenever there are people who... there is an ethnic group which ..." can change the meaning of our quantifiers so as to ensure that these conditionals will express truths.

A Quantifier Variance explanation of mathematicians' freedom promises to let us satisfy all of the desiderata mentioned above. This view avoids commitment to an apparently unmotivated distinction between acceptable and unacceptable posits. For it lets us to say that all posits describing logically coherent additions to the mathematical and mundane structures recognized by one's current language[11]. Since Quantifier Variance holds that shifts in mathematical usage changes our language to make certain existence claims timelessly true in all worlds (as discussed above) it also satisfies the desiderata that mathematical truths be metaphysically necessary.

The Quantifier Variance explanation above also honors our desire for a uniform account of the meaning and logical form of existence claims about mathematical objects and grammatically similar existence claims involving ordinary objects. Thus, it lets us directly honor Benacerraf's goal of treating apparently gramatically and inferentially similar talk similarly. For, it allows us to say that a single notion of existence is relevant to claims like "Evelyn is prim." and "Eleven is prime." in any given context (though, of course, future choices may change which notion of existence one's language employs) [12].

With this formulation of Quantifier Variance and its possible role in philosophy of mathematics in place, let us now turn to the main task of this paper: answering objections.

4 Compatibility With Metaontological Realism

First, one might worry that accepting a Quantifier Variantist explanation of mathematicians' freedom commits one to skepticism or antirealism about the project of doing ontology. This concern is understandable because, historically,

[11] Note that there are subtle issues involved in ensuring that not only are all posits coherent but that they don't entail some (potentially false) fact about concrete objects, e.g., if the posits were only coherent with a finite universe. See Field's remarks in [7] for details on this point.

[12] Also note that accepting my Core Quantifier Variance thesis above does not require one to accept that normal English employs verbally different expressions corresponding to at least two different quantifier senses (a metaphysically natural and demanding one and a laxer one). Thus, one is not forced to agree to claims like "composite objects exist but they do not really exist" in certain contexts.

Thus, with regard to any particular context we can fully accept David Lewis' point that, "The several idioms of what we call 'existential' quantification are entirely synonymous and interchangeable. It does not matter whether you say 'some things are donkeys' or 'there are donkeys' or 'donkeys exist'...whether true or whether false all three statements stand or fall together."[13]

versions of the Quantifier Variance thesis have often been used to debunk or deflate ontology. Critics of the project of ontology have traditionally combined the Quantifier Variance thesis above with an idea that there is no most natural, metaphysically insightful or joint-carving way to carve up the world.

Thus, past formulations of Quantifier Variance have tended to fold in an additional *parity claim* to the effect that all these meanings are (somehow) metaphysically on par. For example, David Chalmers characterizes Quantifier Variance as (roughly) the idea that, "there are many candidate meanings for the existential quantifier (or for quantifiers that behave like the existential quantifier in different communities), with none of them being objectively preferred to the other."[3] As this illustrates one can combine the Quantifier Variance thesis above with a rejection of ontology by saying that all relevant notions of existence have equal status. One can also reject ontology by combining Quantifier Variance with the claim that there are multiple equally metaphysically natural quantifier meanings or (even) that there is an infinite descending chain of progressively more natural ones[13].

However, as Ted Sider has helpfully emphasized[20], there's no problem with combining Quantifier Variance with realism about ontology, if you think there's a single most natural quantifier sense. For, you can say that the ontologist hopes to investigate what exists in this maximally natural sense. And you can further say that this maximally natural sense can come far apart from the sense used outside the metaphysics room – so that paradigmatic and infamous ontological questions like 'But are there *really* holes?' (i.e., does the maximally natural

[13]For example, forms of Quantifier Variance have been explicitly formulated and endorsed by Amie Tomasson (who wants to use it to account for certain aspects of our knowledge of ordinary objects and debunk metaphysics) [22] [23] and Eli Hersh (who wants to use it to debunk metaphysics) [10]. Both philosophers use Quantifier Variance to debunk metaphysics by arguing that the best Davidsonian charitable interpretation of metaphysicians who seem to disagree about whether there are holes/shadows/meriological fusions etc takes each to be speaking the truth in their own language (using different quantifier senses)[check!]. And Matti Eklund has plausibly argued that this Carnapian proposal is best cashed out as advocating a form of Quantifier Variance[6].

quantifier sense relevant to the metaphysics room recognize holes) are reasonable. Indeed, I would argue that Quantifier Variance combines particularly well with (Sideran) realism about ontology, since it lets one endorse the intuition that a plumber may be speaking truthfully when he says there is a hole in your sink even if the metaphysical question of whether 'holes' literally exist in the maximally natural sense of existence is still open.

One might worry that combining this Quantifier Variance explanation of mathematicians' freedom with a realist approach to ontology creates pressure to think that mathematical existence assertions are (somehow) ontologically second class citizens[14]. in way that would conflict with the uniformity intuitions noted above. But to whatever extent combining a Quantifier Variance explanation of mathematicians' freedom with Siderian realism about ontology tends to suggests that mathematical objects are ontological second class citizens *also* suggests that holes and countries are ontological second class citizens. Thus, this doesn't require an violation of the kind of uniformity intuitions championed by Benacerraf in [1] and discussed above. On the Quantifier Variance explanation of mathematicians' freedom easy access to existence claims in mathematics is a limiting case of easy access to ontologically inflationary conditionals like 'if physical facts are like ... this then there are holes...' or 'if people are doing... then there is a country which...'. Thus, cities and numbers are being attributed entirely parallel semantics.

5 Analytic mathematical existence claims?

Next, the Carnapian heritage of Quantifier Variance might also inspire one to worry that this view is committed to contentious (or mysterious) Carnapian

[14] I think a more reasonable worry along vaguely these lines is that the metaontologically realist fan of Quantifier Variance runs into trouble with a version of the Quinean Indispensibility argument. I discuss this issue in REDACTED.

assumptions about framework stipulations and the analytically or indubitably of mathematics. However, a closer examination reveals that Quantifier Variance is fully compatible with more relaxed and holistic ideas about how use determines meaning. After all, the only novelty of (this version of) Quantifier Variance is that it takes the ordinary picture about how our use determines the meaning and extends it to the quantifiers. And nothing in this idea relies on Carnapian assumptions about the philosophy of language or forces us to change our view on fraught questions like the analyticity of mathematics.

To dramatize this point, consider Williamson's extreme picture [24] on which having a concept requires accepting sufficiently many of some weighted cluster of propositions involving this concept, but any particular one can be intelligibly doubted by philosophers, even claims like 'All vixens are foxes' or instances of modus ponens. This picture rejects the idea of a non-trivial notion of analyticity, and says that essentially nothing is analytic. Yet this it is perfectly compatible with the Quantifier Variantist explanation for mathematicians' freedom. We just have to say that the web or cluster of mathematicians beliefs about, say, the numbers, has the power to produce a suitable change in quantifier meaning.

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6 Verificationist Collapse?

Next, one might worry that accepting Quantifier Variance commits us to a kind of verificationism, on which it would be impossible for human beliefs in the ideal limit of scientific or mathematical investigation to be wrong. For example, our observation practices associated with the term 'rock' would come out to be more reliable if we were interpreted as talking about *rocks-humans-can-observe* (thus excluding rocks only found deep in the mantle) rather than *rocks*. So one might worry that an ideal charitable interpreter choosing between variant quantifier senses would have to understand us as speaking truly (in the ideal limit of human science) about the former, rather than falsely about the latter. Similarly in mathematics, one might worry that the most charitable interpretation of our arithmetical practices would involve taking us to be talking about a non-standard model of (first order) Peano arithmetic if this interpretation ensured that all our first order arithmetical hunches expressed truths[15].

However, we must note that accepting Quantifier Variance does not create this verificationist worry, but only extends its domain of application. Just as a mad Davidsonian interpreter could always take people to be talking about a non-standard model of the numbers so as to make all their conjectures about formally undecidable questions come out true, they could tinker with how 'witch' or 'spy' applies to people so as to make all our unfalsifiable beliefs about these come out true.

And the Quantifier Variantist is quite free to use the same tools which address the familiar version of this threat to handle their particular version. Just as the mere fact that a Davidsonian interpretation *could* interpret someone to always be speaking the truth doesn't mean they should, the mere fact that Quantifier Variance allows one to change the meaning of the quantifier to introduce new objects doesn't mean that is always the right interpretation. Note that the same reasons we have for interpreting people to be making false non-mathematical claims apply in the mathematical case as well.

6.1 Reference Magnets and Quantifier Variance

To see how this works in more detail, remember one of the most popular responses to the general worry raised above: Lewis' account of sparse properties and reference in *On the Plurality of Worlds*[14]. How do our prospects for avoiding the above worry about Quantifier Variance look if we start with something

[15]Remember that every consistent first order theory has some model that makes it true.

like Lewis' story?

Lewis' account takes the world (and all metaphysically possible worlds) to be fundamentally carved up into objects in some preferred way, but supposes there are *more and less reference magnetic joints in nature* which make certain properties (i.e. possible ways of dividing up the objects at all metaphysically possible worlds) more natural and apt for reference than others (e.g., gold is more reference-magnetic than rather than gold-or-feldspar). Accordingly, when your core beliefs and observation/proof procedures of some property expression "P(x)" come out equally truth preserving when interpreted to apply to either class, the more intrinsically eligible one wins out. There is also supposed to be a sliding scale of naturalness, so that you can introduce not-very-natural kinds like vegetable, but still be undetectably wrong about how they apply.

But can't the Quantifier Variantist provide an obvious analog to Lewis' story, with approximately equal plausibility? For the Quantifier Variantist is free to say that some variant quantifier meanings are more eligible than others, just as Lewis says that some property terms are more eligible than others. At most, if they are a metaontological antirealist[16] they may be committed to saying there is no maximally fundamenta/joint-carving/eligible quantifier meaning. But this does not prevent them from saying that quantification over *rocks* is more natural than quantification over *observable rocks* as in the example above.

One should further note that to employ this strategy we don't need to understand *particular mathematical structures* (like the natural numbers) as being reference magnetic. Another possibility is that the interpretation of the logical vocabulary is reference magnetic (e.g. second order quantification over 'all possible subsets' specified first order collections). Such a view has the advantage of putting all second order structures on an even footing. I personally think the

[16]That is, philosophers who combine my quantifier variance explanation part of a more traditional Carnapian program of suspecting or debunking the project of traditional ontology.

latter type of story is more attractive and discuss an example of how to flesh it out in the appendix.

7 Theologians and Yetti Hunters

Finally, even if you accept that Quantifier Variance is (in general) compatible with our being undetectably wrong, a more targeted version of the worry that accepting this explanation makes knowing existence facts too easy can remains. Specifically, one might wonder whether the Quantifier Variance explanation of mathematical existence claims also entails the truth of sentences like "God exists" or "Yetis exist." Given the robust inclinations of many speakers to accept these sentences what differences between these domains and that of mathematics can explain this different treatment given that it doesn't seem to merely be a matter of the strength of the relevant reference magnets[17]? I suggest that what warrants us in interpreting mathematicians to be using a more generous notion of existence, while not interpreting theists to be doing the same, is the fact that mathematicians aren't inclined to view themselves as disagreeing with those who acknowledge the existence of new mathematical structures while theologians would view someone who believed in a different number of gods than they did as being in error.

7.1 Disagreement Behavior

Let me do more to indicate the kind of behavior I have in mind. We noted in section 2.1 that mathematicians are inclined to treat people who seem to be

[17] While one could, in theory, try to explain this difference merely as a result of mathematical structures being weak reference magnets while 'God' and 'Yettis' being strong magnets intuitively this isn't very satisfying. The standard model of the natural numbers seems to be at least as reference magnetic as the concept of Yetti (if not God) and it intuitively seems like the difference in treatment can be traced back to something about our language use not some metaphysical view about how reality is carved up.

positing the existence a different (but logically coherent) collection of mathematical structures as speaking truly. In contrast, traditional theists seem to understand avowals of atheism to be falsehoods rather than making truthful statements using different linguistic conventions. Indeed, it would be absurd to suggest that missionaries are filled with a zeal for converting others to the mere linguistic flavor they prefer in religious discussions. Thus, our usual norms of linguistic interpretation counsel strongly against understanding the content of "God exists" in a way that is compatible with a materialist (among other things) worldview. In contrast, (as dramatized by the quotes in section 2.1), mathematicians don't think that other mathematicians who quantify over more (logically coherent) mathematical structures are making a mistake about which mathematical objects exist. I think this explains why the proponent of Quantifier Variance can say that variant theological doctrines are wrong whereas variant mathematical practices would be right[18].

Least this appeal to rationalizing a kind of 'disagreement behavior' as a component in charitable interpretation seem ad hoc, we should note that exactly analogous point has emerged independently from recent debates on recent debates in meta-metaphysics. For instance Sider (among others) [21] responded to Hersch's [10] suggestion that ontological debates were just verbal disputes by bringing up this kind of disagreement behavior[19].

I am similarly suggesting that theologians' tendency to treat apparent dis-

[18]This is not to say that *no* people who claim to believe in God have easy knowledge of some foundational existence claims in the same way that (I claim that) mathematicians and sociologists do. There are some people (unitarians, priests whose livelihood depends on their accepting a particular form of words and C.S. Lewis' vicar who, "has been so long engaged in watering down the faith to make it easier for a supposedly incredulous and hardheaded congregation that it is now he who shocks his parishioners with his unbelief, not vice versa. "[12]) who will say that all religions express the same deep truths -which are also known by reverent and moral (self described) atheists - in different ways. I don't think it would be bad to say that such mellow 'sophisticates' about God talk can count as speaking the truth and having easy knowledge of whatever they express by "There is a God", though figuring out how exactly to interpret them can be an involved and sometimes perplexing matter.

[19]Compare my account of the relationship between disagreement behavior and natural kinds below with Sider's discussion of treating some expression 'as a theoretical term' in that paper.

agreement with logically coherent atheists and one another over what gods exist as substantive and core to religious practice makes it charitable to interpret them as genuinely disagreeing with one another. This contrasts with mathematicians' casual attitude towards apparent disagreement over what mathematical structures exist makes it natural to translate variant logically coherent mathematical practices as simply using slightly different quantifier senses.

7.2 Relationship to reference magnetism

My view about the role of dispositions to disagree in charitable interpretation fit naturally into the picture of words latching on to proof-transcendent natural kinds evoked in the previous section as follows. To the extent your practice attempts to firmly latch on to a natural kind you will expect people with a sufficiently similar practice to latch onto the same natural kind. When people (like the atheists and theists) judge themselves to be in disagreement this reflects a judgment that their practices are similar enough that they are making conflicting claims about the same natural kind.

Conversely, to the extent that you don't expect some facts about how your words apply to track a reference-magnetic natural kind, you will treat people with behavior that varies from yours as just meaning something different by their words. Because chemists want and expect 'gold' to track a very natural kind, they will treat people who seem to say more different things about what gold is as having a genuine disagreement with them. To the extent that we expect 'martini' to track a much less reference magnetic natural kind, we will allow that there may just be different notions of martini and discussion about whether sweet drinks in a martini glass are martinis may be merely a verbal dispute.

So I'd like to suggest the following crude model. Charitable interpretation attempts to balance understanding someone as latching on to natural kinds with making their beliefs come out true. But the more a person is disposed to treat variant practices as genuinely disagreeing, the more heavily reference to natural kinds should be weighted in this translation. And the more a person is disposed to treat variant practices as not really disagreeing, the more charitable it is to make these beliefs come out true, even at the cost of making this aspect of how their concept relates to the world somewhat arbitrary.

Mathematics presents a nice mixed case. I think that mathematicians behave like their practice lets them latch on to the reference magnetic target of meaning 'really all possible subsets' and a reference magnetic notion of logical coherence, but not additional reference magnetic facts about which logically possible mathematical structures are realized[20].

To the extent that theologians treat people who seem to agree with them on logical coherence facts as still disagreeing, it is appropriate to treat them as attempting to track a further natural kind beyond logical possibility and the ability to mean all possible subsets. A similar point applies to eccentrics who believe in Yettis. Yetti-chasers think that they mean the same thing as (apparent) non-believers by “animal”, “furry” etc. and act like they genuinely disagree with people employing these variant practices. Thus, we cannot charitably reconstruct their assertions and behavior by imagining that they started with agreed on beliefs about non-Yetti objects, and then stipulatively introduced existence claims about Yettis so as to make ‘Yettis are furry animals’ come out true (even at the cost of shifting their quantifier meaning and expanding the application of terms like 'furry' and 'animal').

[20]In plumbers' talk of holes, we also have a mixed case. Plumbers' talk of holes definitely aim at tracking ideal-science-transcendent facts, and hoping that ones hole talk will latch on to a natural kind. But they take these natural joints in reality to be the same (or a portion of) those which we latch onto in talking about how matter is diffused through space. So they don't treat apparent disagreements about how hole facts supervene on matter facts as significant.

8 Conclusion

In this paper I have argued that a form of moderate Quantifier Variance both is both naturally motivated and urgently needed to do certain work in the philosophy of mathematics (such as accounting mathematicians' freedom to stipulate). Moreover, this view can be readily defended against a variety of common objections.

A On Grasping Powerful Logical Vocabulary

To see how the considerations about reference magnetism in section 6.1 can be fit into an attractive story about how we can latch onto suitably powerful logical notions (like the notion of all possible subsets of some antecedently specified collection) - and thence ruling out nonstandard models for your number talk might work - consider the following story (loosely inspired by McGee's discussion in 'Learning Mathematical Concepts'[17]).

McGee draws attention to our disposition to accept all instances of the induction schema even under conditions of ignorance about how properties are supposed to apply in all extensions of our language which add new vocabulary. He suggests that these inference dispositions can explain our ability to grasp concepts like the intended structure of the natural numbers (i.e., the concept of being an ω-sequence).

Now I propose that our disposition to apply the least number principle (i.e. accept induction) even for sets whose members are specified by some Quantum Mechanical random process could work to push us within range of the reference magnet for *all possible subsets* in regards to our second order quantification over the numbers as follows.

Counter-Inductive Coin Flips A series of un-entangled quantum mechanically random coinflips will take place. The coins correspond to the numbers in a 1-1 order-preserving way. That is: for each natural number n, there is a unique nth coinflip, and the $n+1$th coinflip happens immediately to the right of the nth coinflip and every coinflip has a number. The 0th coinflip will land heads and for each number n, if the nth coinflip lands heads than the $n+1$th coinflip will land heads BUT not all these coinflips will land heads.

We think the above claim is both actually false and physically impossible. If all combinations of un-entangled quantum mechanically random events are physically possible (as seems plausible), then making the claim that 'it is not physically possible that Counter-Inductive Coin Flips' come out true (while interpreting talk of physical possibility etc. in a standard way) requires taking us to be talking about something which satisfies the full second order axiom of induction, rather than any of the non-standard models of our usual first order axioms for the numbers, PA, since these non-standard models combine a genuine ω sequence with extra stuff and have a structure like: $0, 1, 2... -2*, -1*, 0*, 1*$ ...[21]. And similarly , one might argue that our mere acceptance of ¬Counter-Inductive Coin-Flips (without any appeal to physical necessity) in advance of any information about the coins is best rationalized by taking us to mean the standard model of arithmetic (which satisfies full second order induction), rather than one which merely satisfies induction for all properties which are describable in some fixed mathematical language[22]

More generally, (I think) one can think of the same kind of dispositions McGee invokes (our willingness to make certain inferences about mathematical structures in ignorance of physical facts and our dispositions to keep them

[21] And such models would seem to make it physically possible for the scenario with coinflips to be realized.

[22] Of course, one could always perversely interpret us as meaning a model which satisfies induction for all English language predicates *as they will apply in the actual world*, but this is where appeal to differences in relative naturalness/reference magnetism of logical vocabulary come in.)

through various various expansions of our language) as providing our grip on - something with the expressive power of - second order quantification over any arbitrary collection of base objects (i.e., to talk about ‘all possible ways of choosing’ some cats, some politicians, some mereological fusions of physical objects etc,), which then lets us pin down the intended behavior of various specific mathematical structures (including the natural numbers) indirectly.

To summarize, there seem to be fairly natural and attractive stories to tell about how our use could help us latch on to a reference magnetic notion with the power of second order quantification or onto particular intended structure. Admittedly these stories will not satisfy the determined Putnamian skeptic. But neither will more familiar Lewissian appeals to reference magnetism which are commonly used to resist unattractive verificationism about how properties like ‘gold’ apply within some domain of objects.

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