Conservativity (or something like it), Logicism and Mathematicians' Freedom

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I take the following views to have wide appeal

- Mathematicians' freedom: Mathematicians are (in some important sense) free to introduce almost any logically coherent axioms they like.
    - considering this fact can help answer access worries, by reducing
        - traditional access worries about knowledge of mathematical objects
        - to access worries about knowledge of logical coherence
- Weak Logicism: There's a close relationship between math and logic

Quantifier Variance approaches develop the above ideas by saying...

- Adopting almost any logically coherent pure mathematical axioms would implicitly redefine terms (including the quantifiers) so they expressed truths.
  - (without all quantifier meanings having to be restrictions of some most natural maximal one)
- So mathematical knowledge can be rationally reconstructed as got from logical knowledge and stipulation alone.

I like the QV approach because it promises to avoid:

- nominalists' worries re: treating math talk very differently from shadow and restaurant talk.
- bad company worries re: commitment to a unique right choice between pairs of internally coherent but incompatible axioms
  - xhearts and xkidneys
  - full set theory and full mereology

  ,
- arbitrary stopping point worries for plenitudinous platonism (e.g. why doesn't the heirachy of sets not go up higher)

However the QV approach faces familiar challenges about:

- Why can't we similarly account for
  - biological or
  - (claimed) theological knowledge

  by appeal to logic and stipulation alone?
- Exactly what kind of logical knowledge needed is needed to recognize acceptable mathematical posits?
- Why is any logical knowledge necessary, given that (in a sense) you can stipulate anything, even syntactic inconsistencies? (c.f. Warren[**?**])

So it would be nice to have a clearer, philosophically motivated, account of

- **when and how** creating rational reconstructions involving learning by stipulative definition can help answer access worries
- **why** this is the case.

In this talk I'll explore using the notion of conditional logical possibility $\Diamond_{...}$ to provide such an account of knowledge by stipulation.

In previous work ('Modal Structuralism Simplified' *A Logical Foundation For Potentialist Set Theory*) I've argued that we can use $\Diamond_{...}$ to attractively

- formulate potentialist set theory in a way that lets you
    - avoid other formulations' appeal to second order quantification, plural quantification and quantifying in/de re possibility.
    - justify the (potentialist translations of the) ZFC axioms from modal principles and methods that seem clearly true/truth-reserving
- non-paradoxically describe and reason about truth conditions for (certain) sentence in languages more ontologically profligate than our own.

So I thought it might be useful for characterizing learning by stipulation too.

- Warning: This is a much newer, less developed, project.
- I'll be trying to advertise a research project, not give all relevant details.

Specifically, I'll sketch an 'epistemic dynamics' for stipulation, describing how acts of attempted stipulative redefinition can

- give us knowledge - of whatever propositions the postulates put forward as attempted implicit definitions express in our post-stipulation language
- but destroy other epistemically valued language states like
  - ability to pick out (non-logical) natural kinds
  - association of terms with reliable observation procedures

Then I'll argue that this model suggests a principled explanation for why

- the kind of mathematical knowledge we seem to have
  - i.e., mathematical knowledge **alongside certain epistemically valued states** as mentioned above
  - can be rationally reconstructed as got from logical insight and stipulations restricted so as not to damage these good states.
- But claimed biological and theological knowledge cannot be so reconstructed etc.

I'll end by (if time permits) arguing that this approach suggests a crisp, philosophically motivated

- answer to questions about **the limits of mathematicians' freedom** raised by Stephen Mackereth when formulating [**?**] (neo?) neo-logicism
- (slight) correction to Field's influential suggestion that good mathematical posits are conservative*

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To motivate my proposal, consider the following features of mathematical practice.

Ordinary people

- attribute mathematicians significant freedom to introduce new axioms
- allow reasoning from resultant overall mathematical principles to figure in scientific reasoning
- **but** expect doing this won't change meanings in our language overall in such a way that e.g.
  - old textbooks about botany and history have to be rewritten because their sentences no longer express truths
  - (roughly speaking) observation practices of asserting/rejecting non-mathematical claims need to be changed
  - abduction practices of treating concepts as natural kinds have to be changed

Correspondingly, mathematicians try to avoid making posits which would jeopardize this ordinary practice:

- syntactically inconsistent/logically incoherent posts
- posits that otherwise imply/require certain kinds of logically contingent constraints on how non-mathematical terms apply
  - e.g. axioms like those for Boolos' parities that can only be satisfied in a finite universe.

And they regard mathematical axioms that fail in the regard above (e.g., by turning out to let you derive a contradiction) as failing to stipulatively define terms/change the meaning of our language in such a way as to express truths.

In general we take ourselves to have epistemically valuable statuses, which attempted stipulative re-definitions can (in principle) jeopardize.

- (Something like) **observation procedures** for accepting vs. rejecting application of a term when faced with certain input, we'd like to keep using.
- **natural kind terms** we'd like to continue to express something joint-carving and specially suitable for abduction
  - in the sense that short theories using them (rather than grue-some predicates) are more likely to state correct natural laws.
- **analytic-ish/conceptually central sentences and inference rules** for terms, which we expect to keep expressing metaphysically necessary truths/valid inferences (e.g., axioms for set theory with ur-elements)

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Thus, for the purposes of rational reconstruction, it might be helpful to think of us as making stipulations

- empowered to change the meaning of certain antecedntly understood terms (sometimes including the existential and universal quantifiers)
- but not permitted to change other certain aspects of our language, e.g.,
  - the (possible worlds) extensions of certain predicate and relation terms
  - the truthvalue of certain sentences (typically conceptually central quasi-analytic ones)
- and therefore guaranteed to preserve these good statuses.

Attempted stipulations will

- change the meanings of our words so as to make stipulated sentences/inference rules come out true, *if* this can be done compatible with the permissions given
- fail and not change meaning at all, if there is no way to make all the stipulated sentences come out true by making such changes

So, plausibly, making an attempted stipulation (successful or not) that's not empowered to change the possible-worlds extensions of predicates/relations with a valued status, will preserve this valued status (and our warrant to presume it continues).

- e.g. attempted stipulations required to preserve the (cross possible-worlds) extension of
  - 'rabbit' will preserve the reliability of 'rabbit' acceptance/rejection procedures
  - 'gold' will preserve warrant for 'gold' as expressing a natural kind when doing abduction.
  - 'all things considered ought' will preserve the legitimacy of expecting/requiring certain connections between 'all things considered ought' claims and action action

Specifically (for the present talk) I'll take this list of permissions includes

- permission to change quantifier meaning, so as to 'talk in terms of more objects' (or not)
- a list of relation symbols whose (possible worlds) extension can be changed[1]
- sentences whose truth is to be preserved (typically analyticities in the old language)
  - e.g., we'd typically expect the set axioms of extensionality and pairing to remain true when we introducing new kinds of objects.

[1] Here I'll only try to model stipulations that keep FOL inferences valid, so many inferences can be replaced with corresp. material conditionals

Example 1: a traditional explicit definition for bachelor might have

- stipulandum: ‘For all x, x is a bachelor iff x is an unmarried man’
- permissions: can’t change the quantifier, must preserve the extension of all terms other than ‘bachelor’

Example 2: a neo-carnapian stipulative definition introducing the natural numbers might have

- stipulandum: second order peano axioms (or some version of them), all numbers are mathematical objects
- permissions: can change the quantifier meaning, can (re) define number and plus, must preserve the truth of ZFCU.

Example 3: a neo-carnapian stipulation introducing in-cars might have:

- stipulandum: whenever a car is in a driveway therere is an in-car co-located with it; in-cars don't survive exiting driveways etc.
- permissions: can change quantifier meaning, must preserve certain analyticities and can change the extension of 'is located at' and 'incar',
  - caveat: Really I'd like to say this stipulation can only add to the extension of '..is located at..'
    - pairs $\langle x, y \rangle$ where x in the extension of 'incar'
    - (hence must preserve how 'is located at' relates objects of antecedently understood kinds to places...)

Admittedly

- Explicitly listing such permissions would take a long time.
- In real life cases helpfully reconstructable as involving stipulative definition, people just assert something like the stipulandum.
- But perhaps we have unspoken defaults that fill in permissions relevant to stipulations in any given context
  - c.f. Linnebo saying a similar thing about custom setting these defaults in [**?**]
  - c.f. the design principles of Object Oriented Programming for inspiration from
    - how human designed programming languages have approached the engineering problem of setting defaults for language change to make introducing new kinds of objects (epistemically) safe and convenient.

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Now let's turn to thinking about the 'epistemic dynamics' these attempted acts of stipulation

- how do acts of attempted stipulation change our knowledge and other epistemically valued states?

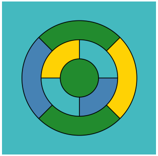
To state this theory (and clarify the kind of logical knowledge needed), I'll use a notion of conditional logical possibility.

- Many have argued for accepting primitively modal notions of logical possibility and necessity ($\Diamond$ and $\Box$), and proof transcendent facts about logical possibility[**?**, **?**].
- When evaluating logical possibility $\Diamond$ we:
  - ignore all limits on the size of the universe
  - consider only the most general combinatorial constraints on how any relations could apply to any objects (c.f. Frege).
  - ignore all subject matter specific constraints on how different relations apply so that, e.g., $\Diamond\exists x(\mathit{Raven}(x) \wedge \mathit{Vegetable}(x))$ comes out true, even though it is metaphysically impossible for anything to be both a raven and a vegetable.

Claims about conditional logical possibility ($\Diamond_{R_1 \ldots R_n} \phi$ 'it's logically possible for ϕ given structural facts about how relations $R_1 \ldots R_n$ apply'.) generalizes this notion by adding one further constraint.

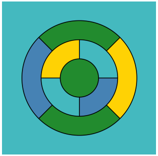
- We only consider to logically possible scenarios that preserve structural facts about how the subscripted relations $R_1, \ldots R_n$ apply.

Here are some motivating examples



It’s logically possible, given the facts about how ‘is adjacent to’ and ‘is a map region’ apply, that each map region is either red,yellow, green or blue and no two adjacent map regions are the same color.

- in this case, just imagine all the teal regions being red



It’s not logically possible, given the facts about how ‘is adjacent to’ and ‘is a map region’ apply , that each map region is either yellow, green or blue and no two adjacent map regions are the same color.

There is no model M which

- is isomorphic to reality in how ‘is adjacent to’ and ‘is a map region’ apply
- makes it true that ‘each country is either yellow, green or blue and no two adjacent map regions are the same color’.

More generally, we could mirror truth conditions for simple conditional logical possibility claims by saying $\Diamond_{R_1,...R_n}\phi$ is true iff there's a model M and function f such that:

- M makes ϕ true
- f is an isomorphism witnesses the fact that M agrees with reality [/whatever scenario is currently being talked about] onthe structural facts about how relations $R_1 \ldots R_n$ apply, i.e.
  - f 1-1 maps the objects related by $R_1, ...R_n$ in reality onto those related by $R_1, ...R_n$ in the M,
  - in a way that respects these relations, e.g., $\forall x \forall y [R_i(x, y) \leftrightarrow \langle f(x), f(y) \rangle$ is in the extension assigned to R_i by M].

It turns out we can use $\Diamond_{...}$ to replace second order quantification in our categorical conceptions of mathematical structures:

For example we can express claims like second order induction:

- **Induct2** $(\forall X)\,[(X(0) \wedge (\forall n)\,(X(n) \rightarrow X(n+1))) \rightarrow (\forall n)(X(n))]$
- **Induct**$_{\Diamond}$: '$\Box_{\mathbb{N},S}$ If 0 is happy and the successor of every happy number is happy then every number is happy.

Note that **Induct** implies that if 0 is green, and the successor of every green number is green, then all numbers are green.

We can use this notion to sharpen the epistemic dynamics for stipulation above.

An act of attempted stipulation is viable iff the stipulated sentence can be got to express a metaphysically necessary truth while only changing the extensions of relations in permitted ways.

- i.e. At each metaphysically possible world w, it's logically possible – without changing the extensions of any terms to be perserved – for the stipulated sentences to be true while all analyticites to be preserved remain true.

That is, if $\blacksquare$ expresses metaphysical necessity and $\Diamond$ logical possibility:
$\blacksquare\Diamond_{RelationsToPreserve}[\text{AnalyticitiesToPreserve} \wedge \text{StipulatedSentences}]$

Results of Stipulation: Whenever a person

- knows or has warrant to presume some act of attempted stipulation is viable i.e., they know the relevant claim
  - $\blacksquare\Diamond_{RelationsToPreserve}[\text{AnalyticitiesToPreserve} \wedge \text{Stipulandum}]$
- Attempts to make this stipulation...

This stipulator will gain

- knowledge of what the stipulated sentence expresses in their new language
- keep warrant for expecting all terms whose possible world extensions they weren't permitted to change to keep any special relationships to the following they once had to
  - observation procedures,
  - connections to action,
  - presumed natural-kind-ness (and hence special status for the purpose of abduction)

Stipulators will also mostly keep knowledge of

- all previously known sentences S which they know (or have warrant for presuming) the stipulation can't change, as
  - $\Box[S \rightarrow \Box_{RelationsToPreserve}[AnalyticitiesToPreserve \wedge Stipulandum \rightarrow S]]$.
  - i.e., that it's logically necessary that if S is actually true then any scenario which preserves all the facts the stipulation is required to preserve must also make S true.

But they may lose

- knowledge of whether various other sentences continue to express truths in their new language,
- warrant for using other relations (which the) previous
  - observation procedures,
  - connections to action,
  - presumed natural-kind-ness for the purpose of doing abduction.

So we face a tradeoff. The more permissions you give an attempted stipulation to

- the easier it is
  - for that stipulation to succeed
  - to know that stipulation is viable (that it can succeed)
- BUT the more power making this stipulation has to destroy (and remove your warrant for presuming) epistemically valued statuses
  - like connections between terms and observation procedures, action, natural kinds.

Note: Strictly speaking, the above picture says recognizing viable stipulation requires knowledge involving metaphysical possibility

- i.e. $\blacksquare\Diamond_{\mathit{RelationsToPreserve}}[\text{AnalyticitiesToPreserve} \wedge \text{Stipulandum}]$
- where $\blacksquare$ expresses metaphysically necessity

But typically we get this knowledge from (almost) logical knowledge alone... by

- inferring from the fact that it's **logically necessary** that it's logically possible (fixing all relevant relations to be pereserved) for the stipulandum and all analyticities to be true
  - $\Box\Diamond_{RelationsToPreserve}[\text{AnalyticitiesToPreserve} \wedge \text{Stipulandum}\ ]$
- to the conculsion that it's **metaphysically necessary** that this is logically possible
  - $\blacksquare\Diamond_{RelationsToPreserve}[\text{AnalyticitiesToPreserve} \wedge \text{Stipulandum}\ ]$

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With this (crude sketch of an) epistemic dynamics for attempted stipulation in mind, let's return to our motivating questions.

First, why think math knowledge can be reconstructed as got from logic and stipulation alone?

- My answer somewhat resembles ‘separate magesteria’ proposals that
  - because pure math claims are somehow specially disconnected from observational vocabulary
  - language change events can make them true without needing to change anything about the application of terms connected to observational practice/science.

But it attempts to make them more principled !

Specifically, claims in math textbooks don't tend to imply (when combined with analyticities) any new constraints on the applications of any terms we take to have the special statuses above i.e.

- vocabulary associated with observation procedures
- (non-logical) natural kind terms
- normative vocabulary grasped partly via a connection to action
  - (e.g., don't accept 'I all things considered ought to ϕ' while unconflictedly intending not to ϕ)
  - c.f. the theory of motivationally grasped concepts in [**?**]

So mathematical knowledge has the unusual feature that

- axioms powerful enough to entail *all* the subject matter claims in textbooks (not just some definitions)
- can learned via attempted stipulations which are (purely logically knowably) viable despite not being empowered to change
  - the extensions of any predicates and relations specially tied to
    - observation proceedures
    - action
    - a non-logical natural kind
  - the truth value of any old analyticities
- and this viability is knowable by logic alone, via learning

In contrast, biology textbooks books do (directly or when combined with analyticities) imply logically contingent constraints on the applications of terms with the three valued statuses above.

Hence any stipulations which

- have a stipulandum powerful enough to imply all the sentences in these textbooks
- aren't empowered to change change the truthvalue of any old analytic sentences or the extensions of any of these
  - observation terms
  - natural kind terms
  - normative terms grasped by connection to action.

can't be recognized as viable by logical insight alone (and generally won't be viable).

So we can't rationally reconstruct biological knowledge as got by logic and stipulation alone.

So we're faced with a dilemma:

- If I imagine starting from nothing and use stipulation to gain knowledge of axioms which imply
  - all the truths in the biology textbook
  - all analyticities in our current language

then I'm left with mystery about my warrant for

- for using observation procedures for certain terms
- assuming that certain terms (mammal, animal, life etc.) express non-logical natural kinds, and giving them corresponding favored status in abduction.

On the other hand, if I start out knowing that I enjoy all the above good statuses ....

Any stipulation which

- I could know (in advance, by logical considerations alone) was a viable
- and attempted to stipulate te truth of an axiom powerful enough to imply all sentences in the biology textbook
- preserved the truth of all analyticities

would have to be empowered to change the extension terms we currently take to be observation and/or natural kind terms.

Hence making this stipulation would destroy my warrant for presuming these good statuses will continue

What about (purported) theological knowledge?

Can we use quantifier variance to reduce acess worries about this? Can we rationally reconstruct this knowledge as got from logic and stipulation alone?[2]? Interestingly, the basic separate magesteria

idea referenced above (c.f. Scanlon?) might suggest we can

- But the (slightly) more fleshed out story I've advocated in this talk suggests we can't.

[2]Thanks to Sylvia Jonas for getting me to think seriously about this question and see her [**?**]

Theological knowledge can seem to constrain the application of antecedently understandable terms that are

- normative concepts motivationally grasped (e.g., a motivationally grasped ‘all things considered ought’)
- non-logical natural kind terms (e.g., consciousness, pain)
- (to a lesser extent) associated with observation procedures
  - e.g. suggestions that justice and compassion often produce material benefits esp. when practiced at national scale

However there's a further sutblty: connections between theological doctrines and the application of antecedenlty understood terms often go through

- ideas about motivated analogy and metaphor rather than explicit conceptually central/quasi-analytic sentences.
  - Ways of unrolling a metaphor can be diverse and debated.
  - But that doesn't mean metaphorical talk imposes no constraint on reality.
    - 'Puglia is the boot on the heel of Italy'
- So one would to plug in some theory of unpacking metaphorical content to flesh out this proposal
  - e.g. maybe accepting a metaphor as apt commits you to the disjunction of all legitimate unpackings of this metaphor.

Overall, we get approx the following picture

- appeals to stipulation/use determining meaning have less power to answer access worries
- the more you interpret religious claims as using antecedently grasped non-religious vocab literally or otherwise imposing strong constraints on how terms with the special statuses above apply
  - e.g., the closer you are to saying that‘man was made in his image’ means G-d literally has arms and legs) the more of an access problem you have.

If you expected no relationship (any logically possible model will do) then you have no access problem.

- But probably few religious people would say this this.

  So an answer to access worries about would probably include something beyond logic and stipulative definitions

- e.g. you could say relevant facts about the application of motivationally grasped concepts are learned via mystical experiences which change/clarify your plans and motivations, experiences
  - unity with everything that reshapes desires re: personal prudence
  - platonic ascent/Joy, interest in something subtle and abstract in comparison with which typical worldly goods seem like mere toys and shadows.

Finally, why can't we rationally reconstruct mathematical knowledge as got from logical knowledge alone?

In principle we have a very wide freedom to change our language and acquire knowledge by stipulation.

- e.g. a sufficiently deterimined person could accept FOL inconsistent axioms and keep the usual FOL inference rules
  - if they were willing to thereby start speaking a language where all sentences expressed truths.
- c.f. Warren on Talking With Tonkers [**?**]

But noting this freedom to stipulate alone can't banish access worries, because

- rationally reconstructing knowledge of mathematical axioms as got by making arbitrary stipulations and holding onto them come what may (even every setnece becomes derivable)
- would leave a mystery about:
    - how we could have gotten math knowledge *alongside other good things* like observation/suprise practices associated with sentences
        - Accept S if you make observation O.
        - Retract S and be surprised if you make observation O'.
    - how we manage to make the kinds of stipulatons that don't require radical change above.

Q: Can’t we instead explain how we avoid language damage by noting our disposition to draw back from the precipice if contradiction is derived (or other conflict with parts of our language practice we like appear)? (c.f. Warren[**?**] and maybe Wittgenstein)

- If we did make a syntactically incoherent stipulation, we’d prevent our language from being trivialized, by doing what we do when we learn about the liar paradox
  - ‘OK I still think most/all normal instances of the T-schema etc. are true. One just needs to be a little cautious about applications of these mostly-good schema to weird cases involving truth predicates’?

A: But (I'd say) merely noting this disposition to damage limitation doesn't solve access worries because

- it can't explain our seeming rational confidence that such damage limitation/repair won't be needed (the liar paradox is an unusual and unexpected case)
- often we damage limit by dropping entire concepts as incoherent and rejecting associated 'analyticities' entirely (not just trimming back as per the liar paradox) esp where there is no obvious/easy trim
  - so if we can't have rational confidence that damage limitation won't be needed
  - we likely couldn't gain knowledge of stipulated would-be-analytic sentences from acts of stipulation .

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Finally, I claimed that this approach helps give a principled (and slightly new) answer to

- some recent questions about which conservatitivty like notion to use when delimiting mathematicians' freedom.

Hartry Field notes that good mathematical posts will likely fail to be conservative[?], since they imply claims about minimum number of objects in the universe

- e.g., PA axioms introducing the numbers will entail that $\exists x \exists y \neg x = y$ ...and corresponding claims that there are >n objects for each n
- These are stateable in FOL with identity, hence statements in our old language.
- not previously derivable (and perhaps not even true).

To fix this Field (in effect) suggests that mathematical axioms just have to be

- conservative* =def adding these axioms doesn't let us prove any new *non-mathematical* sentences (i.e. sentences with vocabulary and quantifiers restricted to non-mathematical kinds of objects).
- This solves the problem above, since PA
  - implies that $\exists x \exists y \neg x = y$
  - but not the corresponding claim with quantifiers restricted to non-mathematical objects.

so it is conservative*

This solution is decently attractive extensionally speaking

- But it doesn't generalize (in any obvious way) to a story about how all other kinds of ontologically inflationary posits work generally
- Hence it can seem to require ad-hoc different treatment to mathematical objects

I've tried to fill in such a more general story in this talk.

- However my story also raises a question also whether Field's conservativity* is slightly too demanding.

On the picture I've suggested we might say: new mathematical axioms don't need to be conservative or even conserviative*

- Given the above story about the functions served by constraints on mathematicians' freedom
  - It's OK (good actually!) to adopt mathematical axioms that let you prove new non-mathematical sentences when these that are already logically necessitated by your old axioms/analyticities (hence already express truths).
    - e.g. new consequences of second order axioms characterizing non-mathematical structures.
  - For, this kind of change can't destroy any of the epistemically valued statuses above.

- Admittedly, situations where this distinction matters might be odd or rare
  - (how could you know something is a second order consequence of your axioms without already being able to prove it?),
  - might involve empirically gained knowledge of second order logical consequences facts as per [**?**]

Relatedly, the proposal in this talk suggests a principled answer to Stephen Mackereth's question in (the talk version of)[**?**]: should logicists regard whether stipulations that are semantically but not syntactically conservative as acceptable?

- If you buy the picture I've been advocating above, only semantic conservativity is needed.

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I'll propose to cash out mathematicians' freedom as a claim that one can rationally reconstruct knowledge of axioms for the purposes of solving access worries, by imagining a process involving

- Starting from logical knowledge (plus other non-matheatmical faculties and epistemically good states accepted by all parties to the dispute)
- gaining knowledge of matheamtical axioms (and conceptions) by acts of explicit stipulation
- *while preserving* other kinds of knowledge and good features of their language and state they expect to be immune to change from mathematical practice
  - c.f. expectation that new mathematical developments won't require us to rewrite biology textbooks

In this talk I have suggested

- the epistemic dynamics of acts of attempted stipulative redefinition (esp. carnapian ones that can change quantifier meaning to get us to talk in terms of new kinds of things)
- a framework for thinking about how much appeals to metasemantic facts can do to solve access worries.

And I've suggested that this framework suggests philosophically motivated way of answering recent questions about how to best understand

- logicism
- mathematicians' freedom

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On Access Worries

I'm thinking of access worries as

- helpfully understood by appeal to
  - *intuitive/informal* coincidence recognition intuitions used in many contexts of scientific and philosophical theory choice
  - norms of ceterus paribus coindidence avoidance
- involving a kind of 'how possibly?' question
  - How could humans possibly have gotten significant accuracy about domain D (e.g. math, ethics) as the realist understands it without benefitting from spooky coincidence (that antirealists about D need not posit)?

And so I think (analogously other ‘how possibly? questions[**?**, **?**, **?**]) access worries can often be attractively answered by providing a **toy model**, a sample explanation which

- may simplify and depart from reality in many ways,
- but preserves all the features that make coincidence banishing explanation seem impossible.

So, in this case, we want a sample explanation/rational reconstruction which explains how creatures

- like us in all ways that make explanation of accuracy seem deeply mysterious
- could have gotten the kind of knowledge of the domain in question we take ourselves to have
- without (intuitively) benefiting from some spooky extra coincidence. [3]

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[3] Note: for these purposes we want an intuitively coincidence reducing explanation.

- e.g., It's not enough buck-passingly explain our acceptance of only true mathematical claims by appeal to our acceptance of only true mathematical axioms and use of truth preserving inference rules.